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TRANSMISSION CHARACTERISTICS OF THE SUBMARINE CABLE.*

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I.

THE transmission characteristics of a conducting system, such as a submarine cable circuit, are determined by its propagation constant, Γ, and characteristic impedance, K, which may be calculated for the frequency $p/2\pi$ from the formulas:

$$\Gamma = \sqrt{(R+ipL)\ (G+ipC)}, \qquad (1)$$

$$K = \sqrt{\frac{R+ipL}{G+ipC}},$$

where R, L, G and C are the four fundamental line parameters, resistance, inductance, leakance, and capacity, all per unit length. These formulas are rigorous for all types of transmission systems; but the determination of the line parameters is not always possible by elementary methods, and may indeed be a matter of considerable complexity and involve rather difficult analysis. In the case of the submarine cable, exact formulas are available for calculating the capacity and leakage and the core impedance. Considerable uncertainty is introduced into the theory, however, on

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account of the lack of a method of determining the "return impedance," that is, the contribution of the "sea return" (sea water, armor wires, etc.) to the effective resistance and inductance of the circuit. An investigation of this problem was undertaken by the writers in connection with the research program of the American Telephone and Telegraph Company and the Western Electric Company.

The purpose of the present paper is to discuss transmission over the submarine cable, and, more particularly, to develop rigorous formulas for the calculation of the impedance of the return conductor of the cable. The results of theoretical calculations are then compared with actual experimental data; and the agreement between theory and experiment is so satisfactory as to indicate that the former is a reliable guide in the design and predetermination of the cable.

Besides providing a method for accurately calculating the transmission characteristics of a submarine cable, the present analysis leads to the following general conclusions:

(1) Contrary to usual assumption, the "sea return" impedance is by no means negligible. Even at quite moderate frequencies there is a considerable crowding of the return current into the immediate neighborhood of the cable, with a consequent rapid increase of the resistance and a corresponding decrease of the inductance of the circuit. Except at the lowest frequencies, therefore, the impedance of the "sea return" is a very important factor.

(2) The armor wires which surround the cable, and which are necessary for mechanical protection, have a very pronounced effect on the impedance of the sea return, and even at moderate frequencies may become the controlling factor. Their action is to screen the current from the sea water itself, and, as the frequency increases, to carry more and more of the return current, until it is almost entirely confined to the armor wires and excluded from the sea water.

(3) The rapid increase in the impedance of the armor wires with frequency, and their pronounced and even controlling effect on transmission makes a thorough-going study of their role in the electrical system a matter of first-class importance. Heretofore they appear to have been regarded only as a mechanical protection, and their effect on transmission has been ignored. The accurate

method of calculating their impedance which is developed in the following pages is believed to have considerable value in this connection.

(4) At relatively high frequencies, the return impedance, and hence the attenuation and the distortion, may be very greatly decreased by a correctly designed thin metallic sheath concentric with the core, and in electrical contact with the armor wires. The very important action of such a sheath, even when extremely thin, does not appear to have been adequately recognized or studied. It is suggested that the introduction of such a sheath affords a means of greatly increasing the range of frequencies which the cable can transmit.

The general problem of determining the transmission characteristics of a system consisting of an insulated conductor surrounded by a concentric ring of armor wires immersed in sea water is of considerable difficulty, since in this case the propagated wave must be represented as a set of component waves centered upon or diverging from the axes of the core and of the individual armor wires. The problem was first simplified by replacing the ring of armor wires by a cylindrical sheath, thus giving circular symmetry to the the structure. The analysis of this case, however, showed that the effect of the iron sheath replacing the armor wires was so pronounced as to make this simplifying assumption of doubtful validity. The general problem was therefore attacked, and rigorous methods developed for calculating the effect of the armor wires upon transmission. The results in this case differ markedly from those obtained for the case of a continuous iron sheath, which indicates that great caution must be used in making assumptions regarding the physical structure of the armoring.

The present paper follows rather closely the course of the writers' investigation. In Section II is analyzed the problem of transmission over a system consisting of n coaxial cylindrical conductors, which may be either in electrical contact at their adjacent surfaces or separated from each other by dielectric spaces. The outermost conductor, consisting of the sea water, is assumed to extend to infinity. This analysis is then applied, in Section III, to the case of a submarine cable which is armored with a continuous iron sheath. This problem is not only of interest in itself, but serves as a first approximation to the case of

an actual cable, and gives a clear qualitative idea of the effect of the various factors on transmission. In Section IV the problem of the submarine cable armored with a ring of iron wires is attacked and solved by rigorous methods, and the theoretical results are then compared with experimental data.

II.

The solution of the problem of transmission of periodic currents over a system comprising n coaxial cylindrical conductors consists in finding the particular solution of Maxwell's equations which satisfies the boundary conditions—continuity of tangential and magnetic forces at the surfaces of the conductors. Let the common axis of the conductors coincide with the Z axis of a system of polar coördinates, R, Φ, Z, and let the electric and magnetic variables involve the common factor $exp(-\Gamma z+ipt)$, Γ is therefore the propagation factor characterizing transmission, and p is 2π times the frequency. This factor will not be explicitly written in any of the work that follows, but it will be assumed to be incorporated in each of the electric variables so that

$$\frac{\partial^n}{\partial z^n} = (-\Gamma)^n, \qquad \frac{\partial^n}{\partial t^n} = (ip)^n.$$

From symmetry, it is evident that the component of electric field intensity in the direction of ϕ vanishes, and that the magnetic lines of force are circles lying in planes perpendicular to the axis of the system, and centered on that axis. Also, the axial and radial electric forces are independent of ϕ. It can be shown that the radial component of electric field intensity *in the conductors* is negligibly small compared with the axial component. The latter, for a given conductor, is of the form $E\ exp(-\Gamma z+ipt)$, where E is a solution of the differential equation

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + (\Gamma^2 - 4\pi\lambda\mu ip)E = 0. \qquad (2)$$

Here λ and μ are the electrical conductivity and the magnetic permeability of the particular conductor, measured in absolute electromagnetic units, and E is a function of r alone.

For the frequencies in which we are interested it may be show that $\Gamma^2/4\pi\lambda\mu p$ is exceedingly small, so that (2) may be written

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} - 4\pi\lambda\mu i p E = 0. \tag{3}$$

We will designate by the subscript j all quantities pertaining to the j^{th} conductor, counting from the axis. The solution of (3) for this conductor may then be written

$$E_j = A_j J_o(\rho_j) + B_j K_o(\rho_j), \tag{4}$$

where J_o and K_o are Bessel functions of zero order, A_j and B_j are arbitrary constants and

$$\rho_j = ri\sqrt{4\pi\lambda_j\mu_j pi} = r\alpha_j.$$

The magnetic field intensity can then be obtained from the curl law,

$$\mu\frac{dH}{dt} = \frac{dE}{dr},$$

which gives

$$H_j = \frac{\alpha_j}{\mu_j ip}\left[A_j J'_o(\rho_j) + B_j K'_o(\rho_j)\right], \tag{5}$$

where the prime indicates differentiation with respect to ρ_j. Taking the line integral of both sides of (5) around circular paths in conductor j lying close to the inner and outer surfaces of the cylinder we obtain

$$A_j J'_o(y_j) + B_j K'_o(y_j) = \frac{2\mu_j ip}{y_j}(I_1 + I_2 + \cdots + I_{j-1}), \tag{6}$$

$$A_j J'_o(x_j) + B_j K'_o(x_j) = \frac{2\mu_j ip}{x_j}(I_1 + I_2 + \cdots + I_j),$$

in which

I_j = current in the jth conductor,

$x_j = \alpha_j a_j$,

$y_j = \alpha_j b_j$,

a_j = external radius of jth conductor,

b_j = internal radius of jth conductor.

The values of the electric field intensity at the inner and outer surfaces of the j^{th} conductor can be written, from (4)

$$E'_j = A_j J_o(y_j) + B_j K_o(y_j),$$
$$E''_j = A_j J_o(x_j) + B_j K_o(x_j).$$

Combining, in turn, each of these equations with relations (6) to eliminate A_1 and B_1, we obtain

$$E'_j = Z'_{j1}I_1 + Z'_{j2}I_2 + \cdots + Z'_{jj}I_j \qquad (7)$$
$$E''_j = Z''_{j1}I_1 + Z''_{j2}I_2 + \cdots + Z''_{jj}I_j,$$

in which

$$Z''_{jk} = 2\mu_j ip \left[\frac{1}{x_j} \frac{J_o(x_j)K'_o(y_j) - J'_o(y_j)K_o(x_j)}{J'_o(x_j)K'_o(y_j) - J'(y_j)K'_o(x_j)} \right.$$
$$\left. - \frac{1}{y_j} \frac{J_o(x_j)K'_o(x_j) - J'_o(x_j)K_o(x_j)}{J'_o(x_j)K'_o(y_j) - J'_o(y_j)K'_o(x_j)} \right], \quad k \neq j$$

$$Z''_{jj} = \frac{2\mu_j ip}{x_j} \left[\frac{J_o(x_j)K'_o(y_j) - J'_o(y_j)K_o(x_j)}{J'_o(x_j)K'_o(y_j) - J'_o(y_j)K'_o(x_j)} \right],$$

$$Z'_{jk} = 2\mu_j ip \left[\frac{1}{x_j} \frac{J_o(y_j)K'_o(y_j) - J'_o(y_j)K_o(y_j)}{J'_o(x_j)K'_o(y_j) - J'_o(y_j)K'_o(x_j)} \right. \qquad (8)$$
$$\left. - \frac{1}{y_j} \frac{J_o(y_j)K'_o(x_j) - J'_o(x_j)K_o(y_j)}{J'_o(x_j)K'_o(y_j) - J'_o(y_j)K'_o(x_j)} \right], \quad k \neq j$$

$$Z'_{jj} = \frac{2\mu_j ip}{x_j} \left[\frac{J_o(y_j)K'_o(y_j) - J'_o(y_j)K_o(y_j)}{J'_o(x_j)K'_o(y_j) - J'_o(y_j)K'_o(x_j)} \right].$$

We have now succeeded in expressing the electric forces in the conductors as linear functions of the currents $I_1 \ldots I_n$, the coefficients being of the nature of impedances, by a method which is simply an application of the principle of continuity of magnetic field intensity. The remaining boundary condition, continuity of the tangential component of electrical field intensity gives, where two consecutive cylinders are in electrical contact,

$$E'_{j+1} - E''_j = \left[Z'_{j+1,1} - Z''_{j,1} \right] I_1 + \cdots + \left[Z'_{j+1,j} - Z''_{j,j} \right] I_j + Z'_{j+1,j+1} I_{j+1} = 0. \qquad (9)$$

This gives m relations between the n currents of the system, m being the number of contacts between successive cylinders. In the case where the j and $(j+1)$st conductors are separated by a layer of dielectric material, a relation between the boundary values of electric field intensity may be obtained as follows:

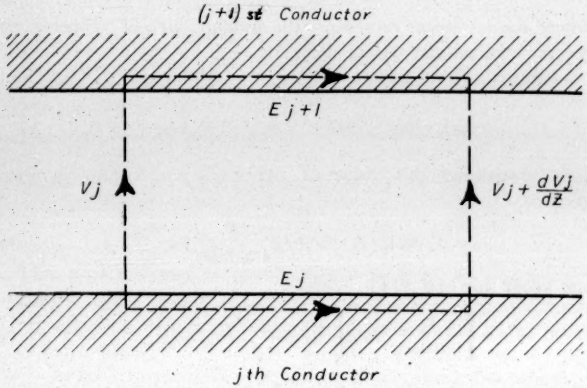
If E_r is the radial electric field intensity in the dielectric, then

$$V_j = \int_{a_j}^{b_{j+1}} E_r dr$$

is the potential difference between the j and $(j+1)$st conductors, in the sense employed in ordinary circuit theory. If we now apply the law

$$\text{curl } E = -\mu \frac{dH}{dt}$$

FIG. 1.



to the elementary contour shown in Fig. 1 we get

$$-\frac{\partial V_j}{\partial z} + E'_{j+1} - E''_j = \mu i p \Phi_j, \tag{10}$$

or

$$\Gamma V_j + E'_{j+1} - E''_j = \mu i p \Phi \tag{11}$$

where Φ_j is the magnetic flux threading the contour and is given by

$$\Phi_j = 2(I_1 + I_2 + \cdots + I_j)\log \frac{b_{j+1}}{a_j}.$$

From the law

$$\text{div } kE = 4\pi Q,$$

$$E_r = \frac{2}{k_j r}(Q_1 + Q_2 + \cdots + Q_j)$$

where Q_j is the charge on the j^{th} conductor and k_j is the dielectric constant of the medium, whence,

$$V_j = \frac{2}{k_j} \log \frac{b_{j+1}}{a_j} (Q_1 + Q_2 + \cdots + Q_j). \tag{12}$$

Furthermore, the rate of gain of charge is

$$\frac{\partial}{\partial t}(Q_1 + Q_2 + \cdots + Q_j) = -\frac{\partial}{\partial z}(I_1 + I_2 + \cdots + I_j) - 4\pi (Q_1 + Q_2 + \cdots + Q_j) g_j / k_j, \tag{13}$$

where the last term represents the leakage current, g_j being the specific conductivity of the dielectric.

From (13) we have

$$\frac{1}{k_j}(4\pi g_j + ipk_j)(Q_1 + Q_2 + \cdots + Q_j) = \Gamma (I_1 + I_2 + \cdots + I_j)$$

and substituting this value of $(Q_1 + Q_2 + \ldots + Q_j)$ in (12) gives

$$V_j = 2(I_1 + I_2 + \cdots + I_j) \frac{\Gamma}{4\pi g_j + ipk_j} \log \frac{b_{j+1}}{a_j}; \tag{14}$$

and from this and (11)

$$-\left[\frac{\Gamma^2}{G_j + ipC_j} - ipL_j\right](I_1 + I_2 + \cdots + I_j) = E'_{j+1} - E''_j \tag{15}$$

where $$G_j = \frac{4\pi g_j}{2 \log \frac{b_{j+1}}{a_j}} \qquad C_j = \frac{k_j}{2 \log \frac{b_{j+1}}{a_j}} \qquad L_j = 2\mu_j \log \frac{b_{j+1}}{a_j}.$$

Subtituting the values of E''_j and E'_{j+1} from (7) in (15) gives

$$-\left[\frac{\Gamma^2}{G_j + ipC_j} - ipL_j\right](I_1 + I_2 + \cdots + I_j) = (Z'_{j+1,1} - Z''_{j,1}) I_1 + \cdots + Z'_{j+1,j+1} I_{j+1} \tag{16}$$

An equation of this sort may be obtained for each layer of dielectric, and these combined with equations (9) and the condition that the electric field intensity in the sea water must vanish at infinity,

$$E'_n = Z''_{n1} I_1 + \cdots + Z''_{nn} I_n = 0,$$

given n relations between $I_1 \ldots I_n$. In order that these shall be consistent, the determinant of the coefficients must vanish.

$$\begin{vmatrix} Z'_{21}-Z''_{11}, & Z'_{22}, & 0 & - - - & 0 \\ Z'_{31}-Z''_{21}, & Z'_{32}-Z''_{22}, & Z'_{33}, & - - - & 0 \\ - & - & - & - - - & - \\ Z'_{j+1,1}-Z''_{j,1}+Z_j, & Z'_{j+1,2}-Z''_{j,2}+Z_j, & - & - - - & - \\ - & - & - & - - - & - \\ Z''_{n1}, & Z''_{n2}, & - & - - - & Z''_{nn} \end{vmatrix} = 0 \qquad (17)$$

where

$$Z_j = \frac{\Gamma^2}{G_j + ipC_j} - ipL_j .$$

This is an equation in Γ^2 of degree equal to the number of dielectric layers, consequently, there are as many independent modes of propagation in the system as there are branches in the network of conductors.

From this point the method of determining the behavior of the system depends upon conditions in the particular problem. For the case where there are k dielectric layers separating the conductors into $k+1$ groups the current on the j^{th} group may be written in the form

$$I_j = A_{j1}\, exp\,(-\Gamma_1 z + ipt) + \cdots + A_{jk}\, exp\,(-\Gamma_k z + ipt)$$
$$+ B_{j1}\, exp(\Gamma_1 z + ipt) + \cdots + B_{jk}\, exp(\Gamma_k z + ipt),$$

where $\Gamma_1^2 \ldots \Gamma_k^2$ are the k roots of the determinant (17) and $A_{j1} \ldots A_{jk}$, $B_{j1} \ldots B_{jk}$ are constants. These constants are not all independent, however, since, for each value of Γ, Γ_1 for instance, there exist k relations of the form (16) which the corresponding set of constants A_{11}, A_{21}, ... A_{k1} must satisfy. The remaining $2k$ independent constants can then be determined from a knowledge of the conditions at the terminals of the conductors.

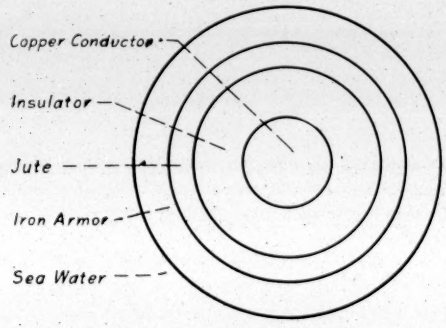
It is important to observe that the transmission characteristics of a system of coaxial conductors are influenced to a great extent by the manner of connecting the various members of the system. Anomalies in the impedance of a complicated network

such as a submarine cable with several conducting sheaths in the return path, may often be traced to lack of proper connections between the sheaths, or to faulty joints.

III.

The submarine cable armored with a continuous coaxial sheath, as shown in Fig. 2, is a particular case of the foregoing, and one which presents a clearer idea of the physical significance of the various steps in the general theory. There are only two

FIG. 2.



groups of conductors, the first consisting of the core conductor, and the second comprising the iron sheath and the sea water, the two groups being separated by the insulating material and the layer of jute. Consequently, there is only one mode of propagation, and the analysis is considerably simplified.

The jute is assumed to contain sufficient sea water so that although it conducts practically no current axially, it maintains equality of potential between the outer surface of the gutta percha and the inner surface of the iron sheath. Consequently equation (10) may be written

$$\frac{\partial V}{\partial z} - E'_2 + E''_1 = -\mu i p \Phi = -ipL_{12}I_1, \tag{18}$$

where E''_1 and E'_2 are the values of electric field intensity at the outer surface of the core conductor and the inner surface of the iron, respectively, V is the potential difference between these two

surfaces, and Φ is the magnetic flux threading unit length of the gutta percha and jute. Also, from (14)

$$-\frac{\partial V}{\partial z} = \frac{\Gamma^2}{G+ipC} I_1 \tag{19}$$

in which I_1 is the current in the core and

$$G = \frac{4\pi g_{12}}{2\log\frac{b}{a_1}}, \qquad C = \frac{k_{12}}{2\log\frac{b}{a_1}}, \tag{20}$$

where g_{12} and k_{12} are the electrical constants of the gutta percha, and b is the external radius of the core. It is evident, that G and C are respectively the leakage and capacity of unit length of the cable. Therefore, from (1),

$$\frac{\Gamma^2}{G+ipC} = R+ipL = Z, \tag{21}$$

where R and L are the resistance and induction of unit length of the cable, including the sea return. Equation (18) may then be written

$$Z I_1 = E''_1 - E'_2 + ip L_{12} I_1. \tag{22}$$

To determine Z we must express E''_1 and E'_2 as functions of I_1.

We have seen that

$$E''_1 = Z_1 I_1, \tag{23}$$

where Z_1 may be termed the "internal impedance" per unit length of this conductor. In fact, when we place $y_1 = o$ in (8) we obtain

$$Z''_{11} = \frac{2\mu_1 ip}{x_1} \frac{J_0(x_1)}{J'_0(x_1)}, \tag{24}$$

which is the usual formula for the internal impedance of a cylindrical conductor.

Similarly

$$E'_2 = -Z_2 I_1 \tag{25}$$

where Z_2 is the internal impedance of the return conductor, the minus sign being due to the fact that the current in the return is in the negative direction of z.

Inserting (23) and (25) in (22) gives

$$Z = Z_1 + Z_2 + ipL_{12}.$$

The quantity Z_2 may be determined in the following manner. From (7) we have

$$E'_2 = Z'_{21} I_1 + Z'_{22} I_2 , \tag{26}$$

where I_2 is the current in the iron sheath. The value of this current can be found by applying the condition of continuity of electric field intensity at the common surface of the iron and the sea water, as in equation (9). This gives

$$Z''_{21} I_1 + Z''_{22} I_2 = Z'_{31} I_1 + Z'_{32} I_2 + Z'_{33} I_3 ,$$

in which I_3 is the current in the sea water. From (8) it can be seen that $Z_{33} = O$, since $x_3 = \infty$, therefore

$$I_2 = \frac{Z'_{31} - Z''_{21}}{Z''_{22} - Z'_{32}} I_1 . \tag{27}$$

Substituting (27) in (26) gives

$$E'_2 = \left[Z'_{21} + \frac{Z'_{31} - Z''_{21}}{Z''_{22} - Z'_{32}} Z'_{22} \right] I_1 ,$$

and by comparison with (25) we have

$$Z_2 = -Z'_{21} - \frac{Z'_{31} - Z''_{21}}{Z''_{22} - Z'_{32}} Z'_{22} \tag{28}$$

as the internal impedance of the return conductor. The resistance and reactance per unit length of this portion of the circuit are then represented by the real and imaginary parts of (28) respectively.

We may then determine R and L from the formula

$$Z = R + ipL = Z_1 + Z_2 + ipL_{12} , \tag{29}$$

where Z_1 and Z_2 are calculated from (23) and (28) and

$$L_{12} = 2 \log \frac{b_2}{a_1} ,$$

b_2 and a_1 being the inner radius of the iron and the outer radius of the core conductor, respectively.

For purposes of comparison, the return impedance is calculated for the case where the iron armoring is absent, the return current being conducted by the sea water alone. As in the preceding case,

$$Z_1=\frac{2\mu_1 ip}{x_1}\frac{J_o(x_1)}{J'_o(x_1)}.$$

The expression for Z_2 simplifies considerably. The electric field intensity in the sea water may be written, from (4),

$$E_2=B_2K_o(\rho_2), \tag{30}$$

the term in J_o being absent in order to permit E_2 to vanish at infinity.

Also, from (6),

$$B_2K'_o(y_2)=\frac{2\mu_2 ip}{y_2}I_1. \tag{31}$$

From (30) and (31) we have

$$E'_2=\frac{2\mu_2 ip}{y_2}\frac{K_o(y_2)}{K'_o(y_2)}I_1. \tag{32}$$

from which the return impedance can be written,

$$Z_2=-\frac{2\mu_2 ip}{y_2}\frac{K_o(y_2)}{K'_o(y_2)}. \tag{33}$$

We have then

$$Z=R+ipL=\frac{2\mu_1 ip}{x_1}\frac{J_o(x_1)}{J'_o(x_1)}-\frac{2\mu_2 ip}{y_2}\frac{K_o(y_2)}{K'_o(y_2)}+ipL_{12}.$$

The resistance and inductance of the sea return of a submarine cable were calculated from formula (28), employing the following values for the constants:

Copper
$$\begin{cases} a_1 = .226 \text{ cm.} \\ b_1 = 0 \\ \mu_1 = 1 \\ \lambda_1 = 6.06 \times 10^{-4} \end{cases}$$

Iron
$$\begin{cases} a_2 = .990 \text{ cm.} \\ b_2 = .737 \text{ cm.} \\ \mu_2 = 100 \\ \lambda_2 = 8 \times 10^{-5} \end{cases}$$

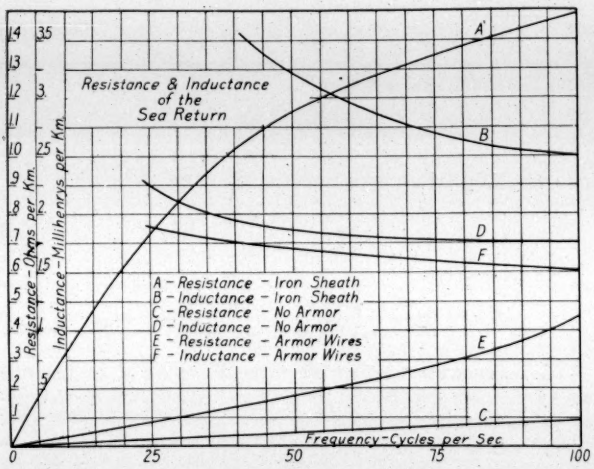
Sea Water
$$\begin{cases} a_3 = \infty \\ b_3 = .990 \text{ cm.} \\ \mu_3 = 1 \\ \lambda_3 = 5 \times 10^{-11} \end{cases}$$

The armoring was then assumed to be replaced by sea water, and the resistance and inductance of the cable were calculated from (33).

The results of the calculations are shown in the curves of Fig. 3.

It is evident from these curves that the effect of the iron armoring is to increase considerably the impedance of the return

FIG. 3.

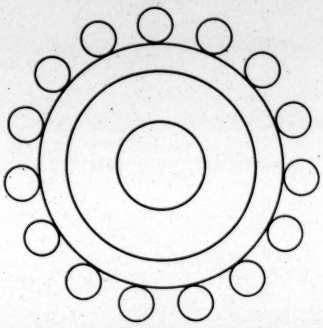

Resistance & Inductance of the Sea Return
Resistance - Ohms per Km.
Inductance - Millihenrys per Km.
A - Resistance - Iron Sheath
B - Inductance - Iron Sheath
C - Resistance - No Armor
D - Inductance - No Armor
E - Resistance - Armor Wires
F - Inductance - Armor Wires
Frequency - Cycles per Sec
0
25
50
75
100

path. The physical explanation of this fact is that the iron acts as a shield to screen from the sea water the electromagnetic effects of the current flowing in the cable conductor. Energy is dissipated in the armoring and is prevented from spreading out through the surrounding medium. The assumption that the armor wires could be replaced by a solid cylinder of iron is, therefore, subject to question, since it is possible that the larger surface area of the assemblage of armor wires, and the gaps between these wires may be effective in diminishing the energy dissipated in the armoring and consequently diminishing the screening effect. This problem is investigated in the following section.

IV.

The physical system under consideration is shown schematically in cross-section in Fig. 4, and consists of an insulated conductor and protective covering of jute, surrounded by a ring of N armor wires immersed in sea water. The method of solution is essentially similar to that given in the preceding pages, and consists in determining the values of electric field intensity at the outer surface of the core conductor and the inner surface of the return conductor, from which the internal impedances of the two conductors can be found.

FIG. 4.



The main difficulty in the analysis is caused by the lack of uniaxial symmetry in the return conductor. This was overcome by employing a method developed by one of the authors [1] in a study of transmission in parallel wires.

The electric field intensity in the sea water satisfies the differential equation

$$\frac{\partial^2 E}{\partial r^2}+\frac{1}{r}\frac{\partial E}{\partial r}+\frac{1}{r^2}\frac{\partial^2 E}{\partial \phi^2}-4\pi\lambda\mu pi E=0,$$

the solution of which is a Fourier-Bessel expansion,

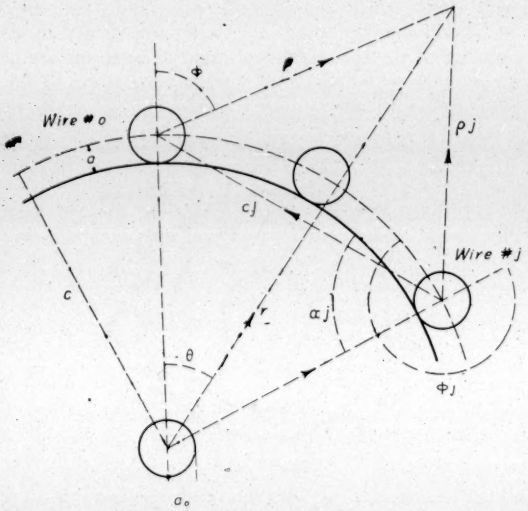
$$E=A_0K_0(r\alpha)+A_1K_1(r\alpha)\cos\phi+A_2K_2(r\alpha)\cos 2\phi+\cdots+,$$

r and ϕ being referred to the axis of the particular wire.

[1] "Wave Propagation over Parallel Wires; The Proximity Effect." John R. Carson, *Phil. Mag.*, vol. xli, p. 607 (1921).

Assuming that the current distribution in the core conductor is independent of the angle ϕ, that is, neglecting the individual character of the armor wires only in their effect on the current distribution in the core, the effect due to the current in the core is

FIG. 5.



represented by the first term of such a series, and the total field intensity may be written

$$E = AK_o(r\alpha) + \sum_{j=o}^{N-1} \sum_{s=o}^{\infty} B_s K_s(\alpha\rho_j) \cos s\varphi_j, \qquad (34)$$

ρ_j and ϕ_j being referred to the axis of wire j, as shown in Fig. 5. That is, the resultant field is expressible as a set of waves centered on the axis of the cable and the axes of the N armor wires.

In the neighborhood of the armor wires the arguments of the Bessel functions are sufficiently small [2] to permit of the approximations

[2] See Note I at end of paper.

$$K_0(\alpha\rho) = K - \log \rho,$$

where

$$K = 0.11593 \log \frac{1}{\alpha},$$

and

$$K_s(\alpha\rho) = \frac{1}{(-\alpha\rho)^s}.$$

The series (34) can, therefore, be written

$$E = A(K - \log r) + B_0(NK - \sum_{j=0}^{N-1} \log \rho_j) + \sum_{j=0}^{N-1} \sum_{s=1}^{\infty} B_s \frac{\cos(s\phi_j)}{\rho_j^s}, \quad (35)$$

in which B_s has absorbed the constant quantities. From this, the magnetic intensity in the sea water can be obtained by differentiation.

Inside any armor wire, at the surface, the field intensities are

$$E = C_0 J_0(\xi) + C_1 J_1(\xi) \cos\phi + \cdots + C_n J_n(\xi) \cos n\phi + \cdots +, \quad (36)$$

$$H_\Phi = \frac{1}{aui\rho} \left[C_0 J'_0(\xi) + C_1 J'_1(\xi) \cos\phi + \cdots + C_n J'_n(\xi) \cos n\phi + \cdots + \right], \quad (37)$$

where $\xi = ai\sqrt{4\pi\lambda\mu\rho i}$,

λ and μ being the electrical conductivity and the magnetic permeability, respectively, of the material of the armor wire. The quantities a and ϕ are centered on the axis of the wire.

In order to determine the coefficients A, B_0, B_1, –, C_0, C_1, – we make use of the fact that the electric and the magnetic field intensities are continuous at the surface of the wire. It is obvious, however, that nothing can be learned by equating (35) and (36) since they are formally dissimilar. We therefore transform [3] the various terms of (35) to a common axis which coincides with the axis of one of the armor wires, hereafter called wire "zero," and the electric field intensity in the sea water, close to the surface of the armor wire, is

$$E = (A + NB_0)K - A\log c - B_0 \log(a c_1 c_2 \ldots c_{n-1}) - \Sigma_0$$

[3] See Note II.

$$+\left[q_1/\zeta-\zeta(A+S_{11}B_o)+\frac{\zeta}{1!}\Sigma_1\right]\cos\phi$$

$$+\left[q_2/\zeta^2+\frac{\zeta^2}{2}(A+S_{22}B_o)+\frac{\zeta^2}{2!}\Sigma_2\right]\cos 2\phi \qquad (38)$$

$$- - - - - - - - -$$

$$+\left[q_n/\zeta^n+\frac{(-\zeta)^n}{n}(A+S_{nn}B_o)-\frac{(-\zeta)^n}{n!}\Sigma_n\right]\cos n\phi,$$

where

$$\begin{aligned}
\Sigma_0 &= S_{11}q_1- S_{22}q_2+ S_{33}q_3\cdots,\\
\Sigma_1 &= S_{02}q_1- 2S_{13}q_2+ 3S_{24}q_3\cdots,\\
\Sigma_2 &= 1.2S_{13}q_1- 2.3S_{04}q_2+ 3.4S_{15}q_3\cdots,\\
\Sigma_3 &= 1.2.3S_{24}q_1- 2.3.4S_{15}q_2+ 3.4.5S_{06}q_3\cdots,
\end{aligned} \qquad (39)$$

$$- - - - - - - - -$$

$$S_{pq}=\sum_{j=1}^{N-1}\frac{\cos pa_j}{(2\sin\frac{j\pi}{n})^q},$$

$$q_n=B_n/c^n, \qquad (40)$$

$$\zeta=\frac{a}{c}.$$

The quantities ϕ and a have the same significance as in equations (36) and (37).

The tangential magnetic field intensity in the sea water at the surface of wire "zero" is, therefore,

$$H_\phi=\frac{1}{ip}\frac{dE_s}{da}=\frac{1}{ipa}\left[-B_o+\cos\phi\left[-q_1/\zeta-\zeta(A+S_{11}B_o)+\frac{\zeta}{1!}\Sigma_1\right]\right. \qquad (41)$$

$$+2\cos 2\phi\left[-q_2/\zeta^2+\frac{\zeta^2}{2}(A+S_{22}B_o)-\frac{\zeta^2}{2!}\Sigma_2\right]$$

$$- - - - - - - - -$$

$$\left.+n\cos n\phi\left[-q_n/\zeta^n+\frac{(-\zeta)^n}{n}(A+S_{nn}B_o)-\frac{(-\zeta)^n}{n!}\Sigma_n\right]- - -\right].$$

To satisfy the condition of continuity of electric and magnetic field intensities at the surface of the armor wire it is necessary that the coefficients of the corresponding terms of (36) and (38) and of (37) and (41) to be equal. This gives

$$C_0 J_0(\xi) = (A + NB_0)K - A\log c - B_0 \log (a c_1 \ldots c_n) - \Sigma_0 , \tag{42}$$

$$C_n J_n(\xi) = q_n/\zeta^n + \frac{(-\zeta)^n}{n}(A + S_{nn}B_0) - \frac{(-\zeta)^n}{n!}\Sigma_n , \quad n = 1 - - \infty \tag{43}$$

$$\xi J_0'(\xi) C_0 = -\mu B_0 , \tag{44}$$

$$\xi J_n'(\xi) C_n = -n\mu \left[q_n/\zeta^n - \frac{(-\zeta)^n}{n}(A + S_{nn}B_0) - \frac{(-\zeta)^n}{n!}\Sigma_n \right] , \quad n = 1 - - \infty \tag{45}$$

From these expressions the quantities $B_1 \ldots C_1 \ldots$ can be determined. Multiplying (43) by $n\mu$ and subtracting (45) gives

$$C_n = \frac{2n\mu q_n}{\zeta^n} \frac{1}{n\mu J_n(\xi) - \xi J_n'(\xi)} , \tag{46}$$

which expresses C_n in terms of q_n. Multiplying (43) by $\xi J'_n(\xi)$ and (45) by $J_n(\xi)$, and subtracting gives

$$q_n = (-1)^n \lambda_n \zeta^{2n} \left[\frac{1}{n}(A + S_{nn}B_0) - \frac{1}{n!}\Sigma_n \right] , \quad n = 1 - - \infty \tag{47}$$

where

$$\lambda_n = \frac{n\mu J_n(\xi) - \xi J_n'(\xi)}{n\mu J_n(\xi) + \xi J_n'(\xi)} . \tag{48}$$

From the infinite set of simultaneous equations (47) the infinitely many varieties q_n may be determined in terms of A and B_0.[4]

We have thus determined the arbitrary constants $C_0 \ldots C_n$ and $q_1 \ldots q_n$ (or $B_1 \ldots B_n$) as functions of A and B_0. It remains to express the latter quantities in terms of physical quantities. If I_1 is the

[4] See Note III.

current in the armor then $\frac{I_1}{N}$ is the current in a single wire. Integrating (41) completely around the armor wire "zero" gives, therefore,

$$2\pi i\frac{I_1}{N}=-B_0\,. \tag{49}$$

Similarly, if I_0 is the current in the core conductor, we find

$$2\pi iI_0=-A\,. \tag{50}$$

We can, therefore, express all the arbitrary constants as linear, homogeneous functions of I_0 and I_1.

To determine the relation between these currents, we have from (42) and (44),

$$CJ_0(\xi)=\frac{ZI_1}{N}\,, \tag{51}$$

where

$$Z=\frac{2\mu ip}{\xi}\frac{J_0(\xi)}{J_0'(\xi)}\,.$$

Substituting (49), (50) and (51) in (42) gives

$$\frac{Z}{N}I_1=-2ip\,(I_0+I_1)K+2ipI_0\log c+2ip\frac{I_1}{N}\log(a.c_1\ldots c_n) \tag{52}$$

$$-(S_{11}q_1-S_{22}q_2+S_{33}q_3-\ldots),$$

from which, since $q_1\ldots q_n$ are functions of I_1 and I_0, the ratio I_0/I_1 can be obtained.

Having shown that the constants A, $B_0\ldots$ of the series (35) are proportional to I_0, we can express the electric field intensity at the inner surface of the return conductor in the form

$$E_2=-Z_2I_0.$$

The computation of Z_2 is facilitated by transforming the terms of (35) to the axis of the core conductor[5] and placing $r=c-a$. We thus obtain

$$E_2=-Z_2I_0=(A+NB_0)K-A\log(c-a)-NB_0\log c-N(q_1-q_2+q_3--)$$

$$+(\text{terms containing }\cos\theta,\ \cos 2\theta,\ \text{etc., as factors}). \tag{53}$$

[5] See Note II.

We have, by applying the curl law to an elementary contour which links the core conductor and the return,

$$\frac{\partial V}{\partial z} - E_1 + E_2 = -ip\Phi_{12}, \tag{54}$$

where

$$E_1 = Z_1 I_o = \frac{2\mu_o ip}{\xi_o} \frac{J_o(\xi_o)}{J'_o(\xi_o)} I_o,$$

$$\Phi_{12} = L_{12} I_0 = 2I_0 \log \frac{c-a}{a_0}, \tag{55}$$

and

$$\xi_0 = a_0 i \sqrt{4\pi\lambda_0 \mu_0 ip},$$

λ_o and μ_o being the electrical constants of the core conductor and a_o its radius. The value given above for Φ_{12} holds only for the contour on which E_2 is independent of the angle θ, that is, when the terms of (53) that contain $\cos\theta$, $\cos 2\theta$, etc., vanish. The value of Z_2 to be used in (54) is therefore determined from

$$E_2 = -Z_2 I_0 = (A + NB_0)K - A \log (c-a) - NB_0 \log c - N(q_1 - q_2 +). \tag{56}$$

As before,

$$-\frac{\partial V}{\partial z} = (R + ipL) I_0, \tag{57}$$

where R and L are the resistance and inductance per unit length of the cable, including the sea return.

We have then from (54),

$$R + ipL = Z_1 + Z_2 + ipL_{12}, \tag{58}$$

from which R and L can be determined.

The process of calculating the resistance and inductance of a submarine cable by the method just described may be summarized as follows:

(1) Determine from (47) the quantities $q_1 \ldots q_n$ in terms of A and B_o, and then in terms of I_1 and I_o by (49) and (50).

(2) Substitute these values of $q_1 \ldots q_n$ in (52) and obtain the ratio I_o/I_1.

(3) Substitute for A, B_o and $q_1 \ldots q_n$ in (56) their values in terms of I_o and I_1.

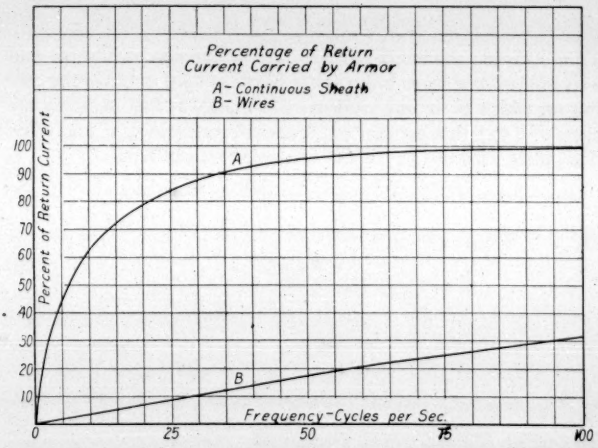
(4) Eliminate I_1 from these two relations, thus obtaining E_2 in terms of I_0. Then $Z_2 = -E_2/I_0$.

(5) Substitute this value of Z_2 and the value of Z_1 calculated from (55) in equation (58).

(6) The resistance and the inductance per unit length of the cable may then be determined from the real and imaginary parts of the latter equation.

The resistance and inductance of a cable of cross-section shown in Fig. 4 were computed by the method just described, the results being given by curves E and F of Fig. 3. The cable in this case is identical with that shown in Fig. 2 previously described,

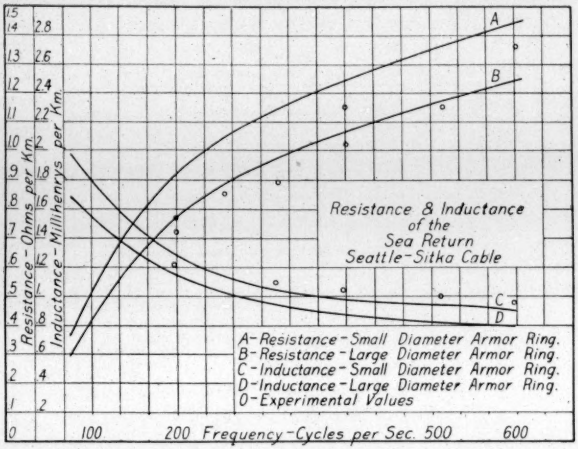
FIG. 6.



except that the continuous iron sheath has been replaced by fifteen wires. The effect of the presence of the iron upon the resistance of the return conductor is still noticeable, although it is much less than in the case of the continuous iron sheath. The reason for this is evident after inspection of the curves of Fig. 6, which show the percentage of return current carried by the armor in the two cases. Especially at the lower frequencies, the return current is much more confined by the continuous sheath than it is by the wires.

As a check of the method, the resistance and inductance of the Seattle-Sitka cable of the United States Signal Corps were calculater for frequencies in the range 50 to 600 cycles per second,

FIG. 7.



and the values so obtained were then compared with the results of measurements recently made upon this cable.[6] The constants used in the calculations were as follows:

Conductor	
Diameter	.216 cm.
Resistance per nautical mile	9 ohms
Rubber Insulation	
Outside diameter	.718 cm.
Capacity per nautical mile	.38 mf.
Armoring	
16 wires	each .242 cm. diameter
Outside Diameter of Cable	2.06 cm.

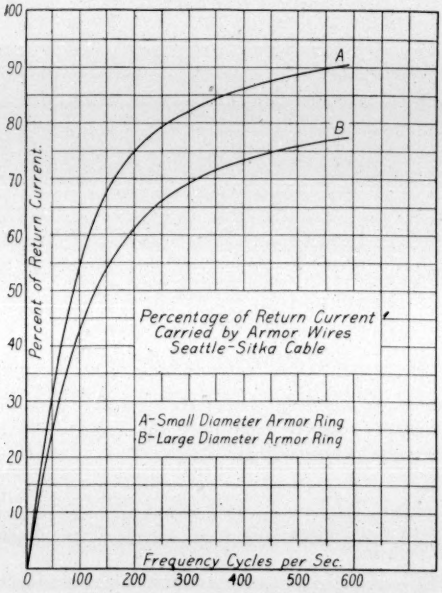
Owing to lack of information concerning the mean radius of the ring of armor wires, two sets of data were computed em-

[6] "The Use of Alternating Currents for Submarine Cable Transmission," Frederick E. Pernot, JOUR. OF THE FRANKLIN INSTITUTE, vol. 190, p. 323, 1920.

ploying the values $c = 0.6148$ and $c = 0.920$, which correspond, respectively, to zero and maximum separation of the armor wires.

The results of the calculations are shown in Fig. 7. The experimental values are indicated by small circles, and agree well with the theoretical values throughout the range of frequencies.

FIG. 8.



The resistance of the sea return increases most rapidly in the region of frequencies used in ordinary telegraphy, 0 to 100 cycles per second. In this range the inductance of the cable also has its greatest values, and these two effects have considerable influence in determining the transmission characteristics of the cable.

The percentage of the return current that is carried by the armor wires is shown in Fig. 8.

CONCLUSIONS.

As was previously pointed out, the effect of the shielding action of the iron armor of a submarine cable is to diminish the electromagnetic field which is propagated through the sea water, and which gives rise to the return current. Combined with this effect is the shielding action of the sea water adjacent to the cable, upon the distant portions. The total shielding effect increases with the frequency until a point is reached where practically the whole of the return current is carried by the armor wires.

Several remedies have been suggested for diminishing the damping effect of the armor wires. It can be proved, for example, that for a given size of core and weight of armor, the number and size of armor wires can be chosen so as to give a minimum value of return impedance. A proper choice of the electrical constants of the material of which the armor is constructed would also be of advantage, since the return impedance is somewhat larger for iron than it is for material of higher or lower conductivity.

Another method of diminishing the return impedance, which has been used in practice, is to wrap the cable core with a number of concentric layers of conducting tape before it is covered with jute. The return current, as it crowds in toward the core with increasing frequency, will then have a path of comparatively low impedance, and at the higher fequencies only a small portion of the current will be carried by the armor wires and the sea water. The impedance of the return path can be calculated for this case by the methods given in the preceding pages. The following table compares the values of the resistance of the return conductor calculated by three different methods, and determined experimentally, for a cable provided with a brass tape 5 mils in thickness.

Resistance of Return Conductor—OHMS per Statute Mile.

Frequency Cycles per Sec.	Approximate Method	Approx. Method* Corrected by Factor $\frac{2}{x}$	Exact Method	Experimental
3000	4.00	3.15	2.87	2.92
10000	4.90	4.25	4.45	4.60

*This is an empirical formula which has been found to be fairly close in most cases. The correction factor suggested itself in that it takes care of the increased surface of the armor wires, as compared with the corresponding continuous sheath.

The experimental values are the results of a series of measurements made by the Department of Development and Research of the American Telephone and Telegraph Company upon the Victoria-Vancouver submarine cable. The calculated values were obtained by both the approximate and the exact methods, discussed in the preceding pages, in which the armor of the cable is treated, respectively, as a continuous sheath and as a ring of wires. The modifications which must be introduced to include the effect of the conducting tape are outlined in the discussion of the general theory. The agreement between the calculated and the measured values of return resistance proves that the method developed in the present paper is accurate even at the highest frequencies employed in telephony.

NOTE I.

NOTE ON BESSEL FUNCTIONS.

The Bessel Functions of zero order of the first and second kinds, $J_0(\rho)$ and $K_0(\rho)$, used in the preceding work are all to a complex argument $\rho = iq\sqrt{i}$ where q is a real number and $i = \sqrt{-1}$. The following formulas may be used for determining the values of these functions:

$q < 0.1$

$$J_0(\rho) = 1 \qquad J_0'(\rho) = -\frac{1}{2}\rho$$

$$K_0(\rho) = \log_e \frac{2}{\gamma\rho} = .11593 - \log_e q - \frac{\pi i}{4}*$$

$$K_0'(\rho) = -\frac{1}{\rho}$$

(*Jahnke u. Emde*, "Funktionentafeln," pp. 97, 98.)

$0.1 < q < 10$

The report of the British Association for 1912 and 1915 give the values in this range of the functions ber q, ber'q, bei q, bei'q, ker q, ker'q, kei q, kei'q which are defined by the relations

*It is to be noted that this approximation for Ko (ρ) differs from the expression used by J. J. Thomson, "Recent Researches in Electricity and Magnetism," p. 264. Thomson's formula (2), from which his approximation was derived, contains a number of errors and should read

$$K_0(x) = (-C + \log 2i - \log x) J_0(x) - 2\,J_2(x) - \frac{1}{2}J_4(x) + \frac{1}{3}J_6(x) \ldots\ldots,$$

where $C = .5772 = \log \gamma$.

$$J_0(iq\sqrt{i}) = \text{ber}\, q + i\,\text{bei}\, q\ ,$$

$$i\sqrt{i}\ J_0'(iq\sqrt{i}) = \text{ber}'q + i\,\text{bei}'q\ ,$$

$$K_0(iq\sqrt{i}) = \text{ker}\, q + i\,\text{kei}\, q\ ,$$

$$i\sqrt{i}\ K_0'(iq\sqrt{i}) = \text{ker}'q + i\,\text{kei}'q\ .$$

$q > 10$

$$J_0(q\sqrt{-i}) = \frac{\varepsilon^{q/\sqrt{2}}}{\sqrt{2\pi q}}\left[\cos\left(\frac{q}{\sqrt{2}} - \frac{\pi}{8}\right) - i\sin\left(\frac{q}{\sqrt{2}} - \frac{\pi}{8}\right)\right],$$

$$J_0'(q\sqrt{-i}) = -i\,J_0(q\sqrt{-i})\,,$$

$$K_0(q\sqrt{-i} = \frac{\varepsilon^{-q/\sqrt{2}}}{\sqrt{\frac{1}{2}\pi q}}\left[\sin\left(\frac{q}{\sqrt{2}} + \frac{\pi}{8}\right) + i\cos\left(\frac{q}{\sqrt{2}} + \frac{\pi}{8}\right)\right],$$

$$K_0'(q\sqrt{-i}) = i\,K_0(q\sqrt{-i})\,.$$

(*Jahnke u. Emde*, pp. 101, 102.)

NOTE II.

TRANSFORMATION OF FOURIER-BESSEL EXPANSION.

In problems involving Fourier-Bessel expansions it is sometimes necessary to transform quantities of the form

$$\frac{\cos s\phi_j}{\rho_j^s}, \frac{\sin s\phi_j}{\rho_j^s}, \log \rho_j\,,$$

from the system of coördinates ρ_j, ϕ_j to the systems ρ, ϕ or r, θ which are related as shown in Fig. 5.

The necessary formula may be derived as follows. We have

$$\frac{\cos s\phi_j + i\sin s\phi_j}{\rho_j} = \frac{\varepsilon^{is\phi_j}}{\rho_j^s} = \left(\frac{\varepsilon^{i\phi}}{\rho_j}\right)^s = \frac{1}{Z_j^s},$$

where Z_j is the conjugate of the vector $Z'_j = \rho_j \varepsilon^{i\phi_j}$. Similarly we may write

$$Z = \rho\varepsilon^{i(\phi - \pi + 2a_j)}\,,$$

$$C_j = c_j \varepsilon^{i(\pi + a_j)}$$

The vectors Z_j, Z and C, as may be seen from Fig. 5, have the lengths ρ_j, ρ and c, respectively, and the directions indicated by the arrows.

By vector addition,

$$Z'_j = Z + C_j$$

whence

$$Z_j = Z' + C'_j,$$

where Z' and C'_j are the conjugates of Z and C_j respectively.

By expansion

$$\frac{1}{Z_j^s} = \frac{1}{(Z'+C'_j)^s} = \frac{1}{C_j'^s}\left[1 - \frac{s}{1}\frac{Z'}{C'_j} + \frac{s(s+1)}{1.2}\frac{Z'^2}{C_j'^2} - \frac{s(s+1)(s+2)}{1.2.3}\frac{Z'^3}{C_j'^3} + - -\right].$$

We have

$$\frac{1}{C_j'^s} = \frac{\epsilon^{is(\pi+a_j)}}{C_j^s} = (-1)^s \frac{\epsilon^{isa_j}}{C_j^s},$$

and

$$\frac{Z'^n}{C_j'^n} = \frac{\rho^n}{C_j^n}\epsilon^{in(2\pi-\phi-a_j)} = \frac{\rho^n}{C_j^n}\epsilon^{-in(\phi+a_j)}$$

Therefore

$$\frac{\epsilon^{si\phi_j}}{\rho_j^s} = \frac{1}{Z_j^s} = \frac{(-1)^s}{C_j^s}\left[\epsilon^{isa_j} - \frac{s}{1}\frac{\rho}{C_j}\epsilon^{-i(\phi-a_j(s-1))} + \frac{s(s+1)}{1.2}\frac{\rho^2}{C_j^2}\epsilon^{-i(2\phi-a_j(s-2))} - \right]$$

Equating the real and imaginary parts gives

$$\frac{\cos s\phi_j}{\rho_j^s} = \frac{(-1)^s}{C_j^s}\left[\cos sa_j - \frac{s}{1}\frac{\rho}{C_j}\cos(\phi - a_j[s-1]) + \right.$$

$$\left. \frac{s(s+1)}{1.2}\frac{\rho^2}{C_j^2}\cos(2\phi - a_j[s-2]) - - - - -\right]$$

$$\frac{\sin s\phi_j}{\rho_j^s} = \frac{(-1)^s}{C_j^s}\left[\sin sa_j + \frac{s}{1}\frac{\rho}{C_j}\sin(\phi - a_j[s-1]) + \right.$$

$$\left. \frac{s(s+1)}{1.2}\frac{\rho^2}{C_j^2}\sin(2\phi - a_j[s-2]) - - - - -\right].$$

Similarly

$$\log Z_j = \log (C'_j + Z')$$

$$= \log C_j + \frac{Z'}{C'} - \frac{1}{2}\frac{Z'^2}{C'^2} + \frac{1}{3}\frac{Z'^3}{C'^3} - \cdots\cdots\cdots$$

$$= \log C_j + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \frac{\rho^n}{C_j^n} \epsilon^{in(\phi - a_j)}.$$

Equating real and imaginary parts we have

$$\log \rho_j = \log C_j + \frac{\rho}{C_j}\cos(\phi - a_j) - \frac{1}{2}\frac{\rho^2}{C_j^2}\cos 2(\phi - a_j) + \cdots +,$$

$$\phi_j = \frac{\rho}{C_j}\sin(\phi - a_j) - \frac{1}{2}\frac{\rho^2}{C_j^2}\sin 2(\phi - a_j) + \cdots +.$$

The following formulas may be derived in a similar manner:

$$\frac{\cos s\phi_j}{\rho_j^s} = \frac{(-1)^s}{C^s}\left[1 + \frac{s}{1}\frac{r}{c}\cos(\theta - \gamma_j) + \frac{s(s+1)}{1.2}\frac{r^2}{c^2}\cos 2(\theta - \gamma_j) + \cdots +\right],$$

$$\frac{\sin s\phi_j}{\rho_j^s} = \frac{(-1)^{s+1}}{C^s}\left[\frac{s}{1}\frac{r}{c}\sin(\theta - \gamma_j) + \frac{s(s+1)}{1.2}\frac{r^2}{c^2}\sin 2(\theta - \gamma_j) + \cdots +\right],$$

$$\log \rho_j = \log c - \frac{r}{c}\cos(\theta - \gamma_j) - \frac{1}{2}\frac{r^2}{c^2}\cos 2(\theta - \gamma_j) - \cdots\cdots.$$

NOTE III.

DETERMINATION OF q_1, q_2, ETC.

The equations (47),

$$q_n = (-1)^n \lambda_n \frac{\zeta^{2n}}{n}(A + S_{nn}B_0) - (-1)^n \lambda_n \frac{\zeta^{2n}}{n!}\Sigma_n, \quad n = 1 - - \infty$$

are linear in the variables $q_1, q_2 -$, since

$$\Sigma_n = n! S_{n-1,n+1} q_1 - \frac{n!}{1!} S_{n-2,n+2} q_2 - - -.$$

The values q_1, q_2 ... may be determined by a method of approximations, q_n being the limit of the sequence

$$q_n^{(0)}, q_n^{(1)}, q_n^{(2)} - - - - -,$$

the successive terms of which are defined by the expressions

$$q_n^{(0)} = (-1)^n \lambda_n \frac{\zeta^{2n}}{n} (A + S_{nn} B_0) ,$$

$$- - - - - - - -$$

$$q_n^{(j+1)} = (-1)^n \frac{\zeta^{2n}}{n!} (A + S_{nn} B_0) - (-1)^n \frac{\zeta^{2n}}{n!} \Sigma_n (q^{(j)}) ,$$

where $\Sigma_n (q^{(j)})$ is the value of Σ_n when q_1, q_2 – are replaced by $q_1^{(j)}$, $q_2^{(j)}$ – – – .

This method, however, while formally simple and direct is not usually well adapted for numerical solution. For all sizes of armor wire and for frequencies of practical importance the argument ζ in the expression (48) is small compared with μ and the quantities,

$$\lambda_1 , \lambda_2 - - -$$

are all nearly unity. This suggests the use of the following method of solution of equations (47).

The solution of the auxiliary set of equations

$$p_1 = -\zeta^2 (A + S_{11} B_0) + \frac{\zeta^2}{1!} \Sigma_1 (p) ,$$

$$- - - - - - - -$$

$$p_n = (-1)^n \frac{\zeta^{2n}}{n} (A + S_{11} B_0) - (-1)^n \frac{\zeta^{2n}}{n!} \Sigma_n (p)$$

in the auxiliary variables p_1, p_2 – may be written,

$$p_1 = -\zeta^2 C_{11} (A + S_{11} B_0) + \frac{\zeta^4}{2} C_{12} (A + S_{22} B_0) + \cdots + ,$$

$$- - - - - - - - - - -$$

$$p_n = -\zeta^2 C_{n1} (A + S_{11} B_0) + \frac{\zeta^4}{2} C_{n2} (A + S_{22} B_0) + \cdots + ,$$

in which C_{11}, etc., are numerics. This solution is effected by retaining a finite number of equations and an equal number of variables and solving by the usual methods. It will be found that

except in extreme cases, a very good approximation can be gotten by ignoring all the p's except the first four. The q's may then be obtained by the relation

$$q_n = p_n + d_n$$

d_n being defined by

$$d_n = (\lambda_n - 1)p_n - (-1)^n \lambda_n \frac{\zeta^{2n}}{n!} \Sigma_n(d).$$

This system is easily adapted to solution by successive approximations,

$$d_n = d_n^{(0)} + d_n^{(1)} + d_n^{(2)} + - - -$$

in which

$$d_n^{(0)} = (1 - \frac{1}{\lambda_1})C_{n1}p_1 + \cdots + (1 - \frac{1}{\gamma_n})C_{nn}p_n ,$$

$$d_n^{(j+1)} = (1 - \frac{1}{\lambda_1})C_{n1}d_1^{(j)} + \cdots + (1 - \frac{1}{\lambda_n})C_{nn}d_n^{(j)} ,$$

C_{n1}, etc., being the numerical coefficients which appear in the expressions for p_1, p_2—.

A very good approximation which holds in most cases is

$$d_n = (\lambda_1 - 1)C_{n1}p_1 + (\lambda_2 - 1)C_{n2}p_2 + \cdots + (\lambda_n - 1)C_{nn}p_n .$$

FEBRUARY 15, 1921.

Compressibility of Aromatic Hydrocarbons. THEODORE W. RICHARDS, EDWARD P. BARTLETT, and JAMES H. HODGES, of Harvard University (*Jour. Am. Chem. Soc.*, 1921, xliii, 1539–1542), find that *solid* benzene at a temperature of 0° C. has a compressibility of 0.0000305 over the range of 100 to 500 megabars, and that *liquid* benzene at a temperature of 20° C. has a compressibility of 0.00007207 over the same range. *Liquid* toluene at a temperature of 0° C. has a compressibility of 0.0000618 over the range stated.

J. S. H.

Studies of Nitroprussic Acid. GEORGE JOSEPH BURROWS and EUSTACE EBENEZER TURNER, of the University of Sydney (*Jour. Chem. Soc.*, 1921, cxix, 1450–1452), have prepared nitroprussic acid and certain of its derivatives. The free acid is obtained as reddish-brown leaflets when an aqueous solution of its barium salt is treated with the calculated amount of sulphuric acid, the precipitated barium sulphate is removed by filtration, and the filtrate is evaporated under diminished pressure. The acid has the formula $H_2Fe(CN)_5NO$; it is readily soluble in water, but the solution is unstable. Its 0.05 normal aqueous solution is approximately 85 per cent. dissociated, as is shown by the conductivity. The acid combines directly with organic bases like pyridine and benzidine to form the corresponding salts. The tetramethylammonium salt has also been prepared. Diethyl nitroprusside is formed by the reaction between silver nitroprusside and an alcoholic solution of ethyl iodide. The ester occurs as red prismatic crystals. Determination of the freezing point of its aqueous solution showed that the ester was dissociated, one molecule of the ester yielding one molecule of nitroprussic acid and two molecules of ethyl alcohol. J. S. H.

The Transformation of Iron at the Curie Point. P. DEJEAN. (*Comptes Rendus,* August 22, 1921.)—By a ballistic method the intensity of magnetization of a stack of carefully turned cylinders of soft steel was investigated as the interstices between adjacent pieces was progressively increased. From the results curves were plotted giving, for each field strength used, the connection between the intensity realized and the interval between neighboring cylinders. This family of curves resembles that obtained in 1895 by Pierre Curie to express the connection of temperature with intensity for varying strengths of field. Interval in one case plays the same rôle as temperature in the other. This analogy leads to the suggestion that it is the progressive separation of the molecular magnets due to elevation of temperature that accounts for the apparent effect of temperature on magnetic intensity. Such progressive separation may be brought about by the gradual transformation of a magnetic form of iron into a non-magnetic form or by an actual, linear separation brought about by the application of heat.

The Curie Point, that is, the temperature at which a magnetic substance ceases to show magnetic properties, rises when the strength of the applied field is made greater. The author ingeniously adduces this fact in favor of his hypothesis that it is in an increase of distances between the magnetic particles that the explanation is to be sought. Were such a particle to lose its magnetic qualities at some temperature, why should it retain them to a higher temperature merely because a stronger field is acting? G. F. S.

THE SHAPE ASSUMED BY A DEFORMABLE BODY IMMERSED IN A MOVING FLUID.*

BY

ENOCH KARRER,

Physicist Applied Science Laboratory, Nela Research Laboratories, Nela Park, Cleveland, Ohio, Member of the Institute.

I. INTRODUCTION.

THE purpose of this article is to discuss from what appears to me to be an entirely new viewpoint, the question of the relation between the shape of an immersed body and the forces acting upon it due to the motion of the fluid. Considerable attention has been given to certain important aspects of this subject. For instance, much work has been done in the determination of the magnitude of the resistance experienced by variously shaped bodies when moving through a fluid at various speeds. This matter is of grave importance in aeronautics.[1] Likewise the distribution of pressure due to motion over bodies of certain shapes has been studied more or less extensively.[2] We may now, however, ask the more general question: What shape will a deformable body assume when immersed in a moving fluid? The behavior of such a body moving in a fluid seems never to have been considered in this regard.

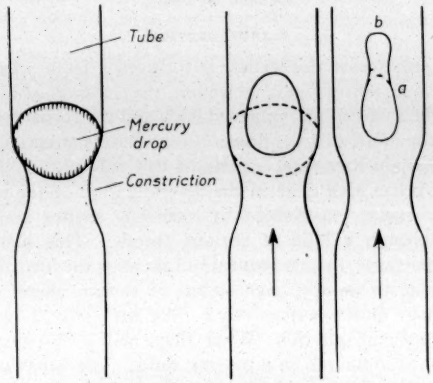
This question was first forcibly pressed into my attention during the exhaustion of the bulb of a mercury arc lamp, when some observations were incidently made on the behavior of a mercury drop. These observations are the only experimental evidence that I am able to put forward at present to substantiate the theme of this paper, that a perfectly deformable body immersed in a moving fluid will assume a shape whose defining contour lines are "stream-lines." No body is, however, perfectly deformable so that in all practical cases, the body will change toward the stream-line shape

* Communicated by M. Luckiesh, Director of the Laboratory.

[1] Reference may be made to the Reports of the National Research Council, Aeronautics Section, for some recent work in this regard.

[2] See Bureau of Standards Scientific Paper No. 394, or *Bul. B. of S.*, p. 489, 1920, for recent data and reference to former results.

until there is equilibrium between the forces of restraint in the body and the forces due to the pressure of the moving fluid. In case of the mercury drop there must be, when the steady state is attained, equilibrium between the forces of surface tension and of pressure. The effect of gravity also can not be ignored in all cases.



The shape of a mercury drop.

A little reflection will bring forth many possible illustrations of the phenomenon in widely separated realms of science—physics, geology, meteorology, and biology.[3]

II. OBSERVATIONS ON A MERCURY DROP.

When a drop of mercury is retained in a tube and a current of air is drawn over it, the shape of it will change. The conditions under which this was first observed are depicted in Fig. 1. A

[3] While searching the literature for suggested illustrations I had the pleasure of very soon encountering the works of Vaughan Cornish, through the kindness of Mr. C. K. Wentworth. (The latter has been interested in the experimental study of the shapes of pebbles. *J. of Exp. Geol.*, 27, p. 507, 1919.) Cornish pointed out after extensive and painstaking observations, that many shapes due to wind and other factors are of definite and uniform type. One cannot help but see some intimation, rather indefinitely expressed in his descriptions and explanations, of the general principle which I am endeavoring to show lies at the bottom of all such forms.

vertical glass tube of about 8 mm. diameter was constricted just sufficiently to retain a rather large drop of mercury. The upper end of the tube was connected to a vacuum pump; the lower end to a mercury lamp. During the exhaustion the mercury lamp was vigorously heated, and as a consequence some of the mercury evaporated and condensed upon the walls of the tube near the constriction, collecting there to form a good-sized drop. Subsequently a small crack developed in the lamp, allowing the air to rush into the evacuated chamber. The stream of air changed the shape of the drop in a most interesting manner. When the air current was not too great the shape of the mercury drop was like that shown in Fig. 2,[4] where the dotted line indicates the shape and position of the drop in repose, and the full line represents the assumed form while the air was rushing by. The drop is elongated in the direction of the air current with the convexity of the blunter end directed against the stream.

When the air stream is allowed to become too great the mercury drop breaks and breaks in a manner that might not be expected, at all events not in a manner that one might be lead to believe from the statements sometimes made as to the breaking of drops. As the velocity of the air becomes greater the globule lengthens, but the lengthening is not entirely at the sacrifice of the general cross-section; rather, the cross-section of the portion of the globule at *a*, Fig. 3, is particularly decreased until the rear portion breaks away. It is impossible to see with the unaided eye the changes taking place in the new-born drop on account of its motion with the stream of gas. It undoubtedly readjusts its shape to conform to the prevailing conditions of streaming. During the formation of it there is also much agitation, and the elongated portion of the drop oscillates violently—suggestive of the existence of unstable conditions there.

This behavior suggests very vividly that the mercury drop adjusts its shape to the conditions of streaming, and as these conditions are dependent largely upon the relative velocity of streaming the mercury drop assumes a different shape for every velocity. It will assume a shape such that the defining contour lines approach steam-lines.

[4] Figs. 1–3 were drawn from memory a few days after the experiments, when the general possible significance of the incidence became convincive.

Is this not indicative of what all deformable or corrodible bodies will do? It is to be expected from the facts that are known regarding the resistance encountered by a body when moving through a fluid. Bodies whose contour lines are stream-lines experience the least retardation. One might expect therefore the inverse phenomenon to take place; namely, that in a body moving through a fluid, there will be set up forces whose directions and magnitudes are such as to cause the body to assume as closely as possible, the shape offering least resistance.

For the sake of brevity this phenomenon will occasionally be referred to as the Principle of *Stream-line Assumption.*

As has already been stated, experimental data are not at hand to be mustered together in order to weigh the generality of the principle. The experiments with the mercury drop gave confirmatory evidence again and again. Whether or not the conditions were too special to allow of any generalization at all can only be decided by further experiments. I will now, however, consider the support which may be found in other realms of science for the generality of the principle of stream-line assumption.

III. OTHER LINES OF EVIDENCE.

a. SHAPES OF SNOWDRIFTS.

Some of the obvious [5] phenomena in which one might expect to obtain evidence of the play of such a principle are in the formation of sand dunes and snowdrifts. In snowdrifts a general characteristic profile has been pointed out by Cornish,[6] who has made numerous and painstaking observations on phenomena of this kind. This profile is depicted by the curve of Fig. 4,[7] the blunt end of which is to windward. This is a most beautiful confirmation of our text. Cornish has named this profile curve the "Eddy-curve." Although this term is connotative of the process of formation of these forms, in that during the formation the curve may be an envelope of numerous eddy currents, yet I prefer

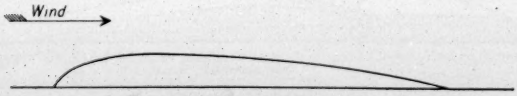
[5] At least most obvious to one who has vivid and delightful recollections of snowdrifts seen during boyhood in and near the Olympic Mountains.

[6] "Waves of Sand and Snow." From these works I am drawing freely for illustrations, and with much appreciation.

[7] This figure and several that follow are taken from Cornish, "Waves of Sand and Snow."

to use the phrase "stream-line" curve or contour. For the latter term refers more definitely to the thing that we are interested in rather than to the process of formation of it. Futhermore, it is the concept of the stream-line contour and its relations to the resistance offered by bodies to moving fluids that has been a guide in arriving at an explanation of shapes of snowdrifts and the like, and in the prediction of the existence of such shapes in

FIG. 4.

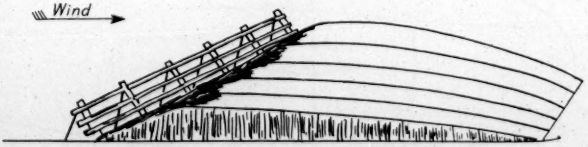


The fundamental curve of snowdrifts. (Redrawn from Cornish.)

certain other cases that had not previously come into my experience.

We may take again from Cornish some excellent examples of the shapes of snowdrifts. In Fig. 5 is shown a snowdrift formed by the obstruction of a fence. In Fig. 6 is a similar one where the fence has become incidental and is no longer the limiting boundary on the windward side. The outline and the cross-section of a snowdrift caused by a clump of bushes are shown

FIG. 5.

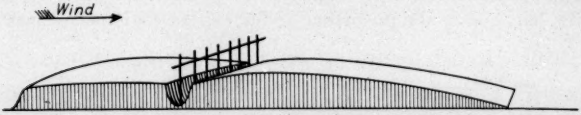


Fence and completed snowdrift. (Redrawn from Cornish.)

in Figs. 7, 8, and 9. A similar case where a house is the incipient interference is shown in Figs. 10, 11, and 12. Another interesting illustration of this type is shown in Figs. 13, 14, and 15, where a round, upright tree causes a hollow to be formed in the drifted snow. This case is interesting because it may be thought of in connection with the experimental work discussed later bearing upon the distribution of pressure over the surface of a round cylinder immersed in moving air. These are illustrations of the formation of barriers rather than of a mere reshaping of bodies. Yet the restraining forces are there, evidenced by the moulding

of these obstacles to the stream-line contour. This shaping lends some assurance that had the drifts been cubical or irregular masses initially, they would, nevertheless, have been shaped to this very same contour.

FIG. 6.

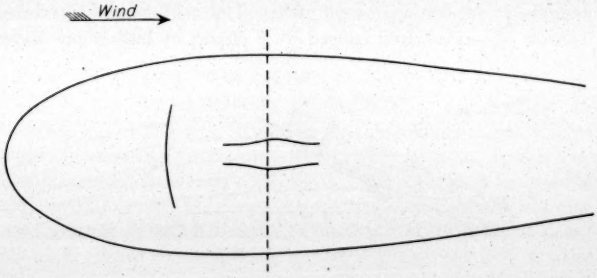


Snowdrift on both sides of a fence; on the right completed, on the left incomplete. (Redrawn from Cornish.)

b. GEOLOGICAL FORMS.

Some examples of the actual shaping of already existing bodies might be looked for in the geological forms. Here, however, there are so many factors of corrosion affecting shape, that it is difficult to select illustrations which show clearly the working of

FIG. 7.



Plan of snowdrift caused by clump of bushes. (Redrawn from Cornish.)

this principle. The forms of erosion, where air or water currents are by virtue of their motion the chief agents of erosion, would present data for decisive judgment. In all actual cases other factors concerned such as the original jointing and lay of the strata, and temperature changes and moisture, may overwhelmingly affect the shape of boulders and the like. The illustrations in Fig. 16 are given without any idea of selection, but are given because they were immediately available.

The so-called "windtables" and erosion pillars may be noted in this connection. Some excellent specimens of the first may be seen in the Petrified Forests of Arizona. The boulders in Fig. 16[8] are at first sight strikingly suggestive of our theme.

The boulders are of granite that is "a coarse-grained, pale red granite composed of quartz, orthoclase, plagiocase and bio-

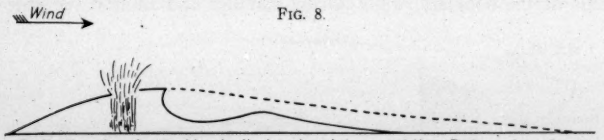


FIG. 8.

Profile (dotted) and central longitudinal section of the same drift. (Redrawn from Cornish.)

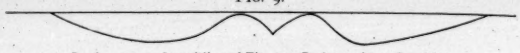


FIG. 9.

Section across dotted line of Fig. 8. (Redrawn from Cornish.)

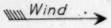
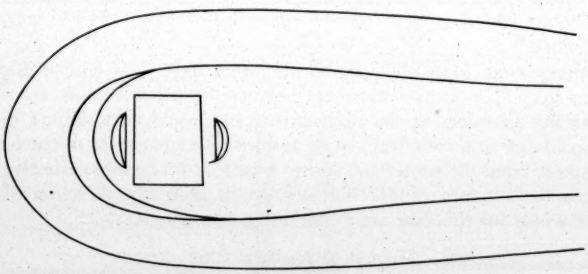


FIG. 10.



Plan of the principal snowdrift around a house on the prairie near Winnipeg. (Redrawn from Cornish.)

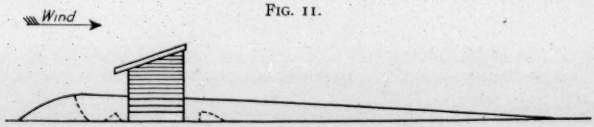
tite in decreasing order of abundance. The average grain size is from 3 to 4 mm., the orthoclase ranging up to 5 mm. long. The height above sea-level at which these boulders occur is 4070 feet

[8] This illustration is taken from Gregory, Australia, *The National Geographical Magazine*, p. 474, December, 1916. For the description (in quotation marks) of them I am indebted to Dr. H. C. Richards, Professor of Geology, University of Brisbane, Brisbane, Australia.

and as they are on the top of a rather bare mountain, they are exposed to rather severe weather conditions. The long axes of these boulders run almost due east and west."

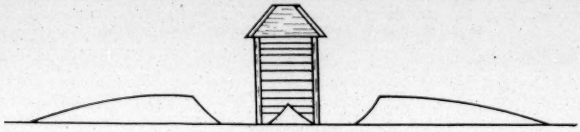
The prevailing winds to which they are exposed are southeasterly in summer and autumn; and westerly in winter and spring. So that during the winter and spring winds blow along the major axis of the boulders, while during summer and autumn the pre-

FIG. 11.



Profile and central longitudinal section of same snowdrift. (Redrawn from Cornish.)

FIG. 12.



Transverse section of snowdrift on lee side of an outhouse on the prairie. (Redrawn from Cornish.)

vailing winds blow obliquely to the major axis. Of course this is a very approximate statement because the winds coming from various directions at the various seasons should be weighted in accordance to their effectiveness as corrosive agents. It might be expected that the winter and spring winds would be more effective than the summer winds. Greater details as to the direction of the winds for this case are given in the following table:[9]

Wind Directions at Stanthorpe (1907–1920).

Season	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.	Calm
	%	%	%	%	%	%	%	%	%
Spring	3	2	11	16	2	8	27	5	26
Summer	3	7	18	25	3	9	14	3	18
Autumn	3	4	16	24	3	6	15	2	27
Winter	2	2	8	12	2	9	22	2	41

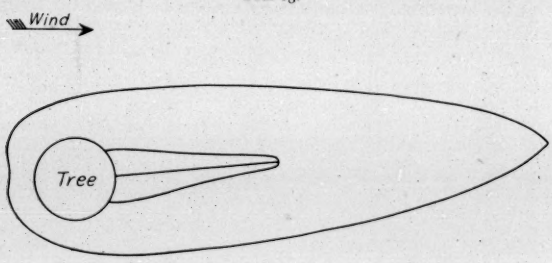
The most striking feature of these winds is that a large per-

[9] Data furnished by Professor Richards, taken from the *Commonwealth Meteorologist.*

centage of them during all seasons have a component east and west; while the total component north and south is relatively small.

Examples of boulders such as in Fig. 16 may be multiplied,[10]

FIG. 13.

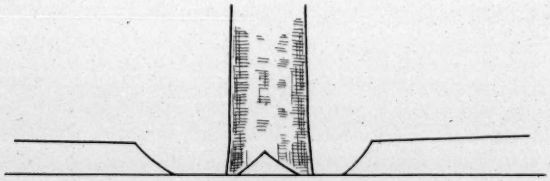


Plan of a hollow around a tree. (Redrawn from Cornish.)

FIG. 14.

Longitudinal section of same. (Redrawn from Cornish.)

FIG. 15.



Cross-section of the same—lee side. (Redrawn from Cornish.)

but for the present not to advantage, for until sufficient data are available they are only suggestive and not illustrative of our theme.

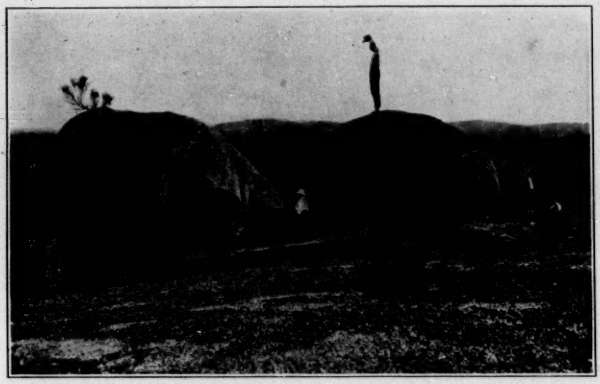
[10] For example, "The Leviathan" and "The Torpedo" and many others in the Buffalo mountains, Victoria, Australia (see *Memoirs of the Geological Survey of Victoria*, 1918, No. 6. The Buffalo Mountains, plates 37 and 40). These were brought to my attention by Professor Richards. They may yield evidence of this phenomenon upon closer analysis of the conditions.

c. OTHER METEOROLOGICAL FORMS.

Very similar to some of these geological forms are the meteorological forms termed "bosses of snow" and "snow mushrooms" pictured in Figs. 17, 18, and 19.[11]

Other meteorological illustrations might be looked for in moving clouds and in fact wherever a portion of the air is made stagnant from some cause or other, such as the "pockets" reported by aeronauts. In observing the clouds for this phenomenon I have seen some excellent specimens of small cumulus clouds, which had a general stream-line appearance and moved with the blunt end forward. In general, however, there appear to be so many factors tending to change the shapes of clouds so rapidly that the number of favorable cases is small. At least this is true in the horizontal section. It is probable that the vertical section would also present the stream-line contour at times, but this viewpoint is seldom to be had.

FIG. 16.



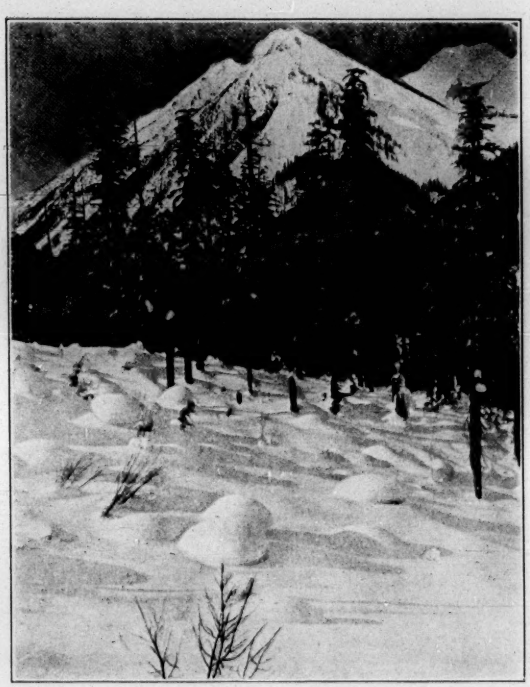
Shape of boulders. (Taken from *National Geographic Magazine*, Dec., 1916, page 474.)

The so-called "pockets" in the air mentioned above might be expected to have a stream-line contour. No data concerning them are available. Very closely related to such pockets are the relatively

[11] Taken from Cornish, "Waves of Sand and Snow."

stagnant portions produced by some obstacle such as a tree, as is illustrated in Fig. 13. This matter will be referred to again when convectional streaming is discussed.

FIG. 17.



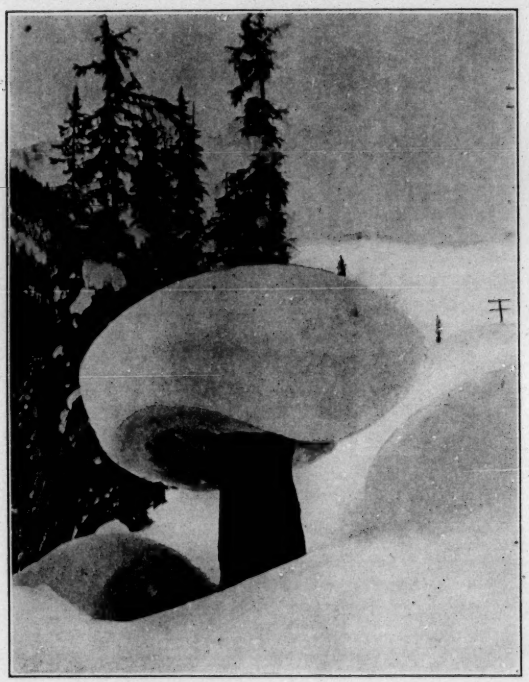
"Bosses" of snow. (Taken from Cornish.)

Still further illustrations may be seen in the so-called snow barchans typified in Fig. 20 and in Fig. 21.[12] These forms may quite frequently be seen on hard roads during cold weather when a light, dry snow is drifting. The raindrop is generally regarded as spherical. It probably is not, for we would expect to find some

[12] Taken from Cornish.

reshaping of it in accord with the principle of stream-line assumption, both for the velocity of falling and for any other relative velocity due to the wind. I had hoped, but now find it impossible,

FIG. 18.



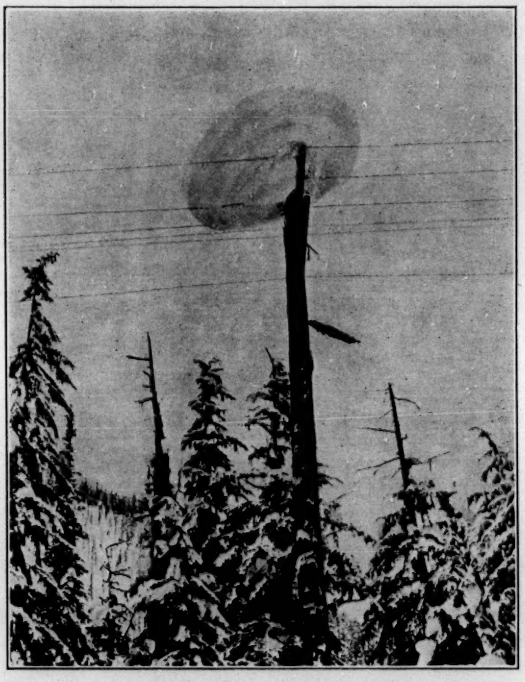
Snow "mushroom." (Taken from Cornish.)

to present data bearing upon this point as well as upon the shape of a falling mercury drop, and upon the manner of breaking up of drops during falling. Icicles too frequently have cross-sectional contours bearing evidence of our theme. Reference will again be made to them when the distribution of forces is considered.

d. BIOLOGICAL FORMS.

Turning now from the realms of geology and of meteorology to the realm of biology, we find illustrations of the working of the principle of stream-line assumption upon the living organism.

FIG. 19.



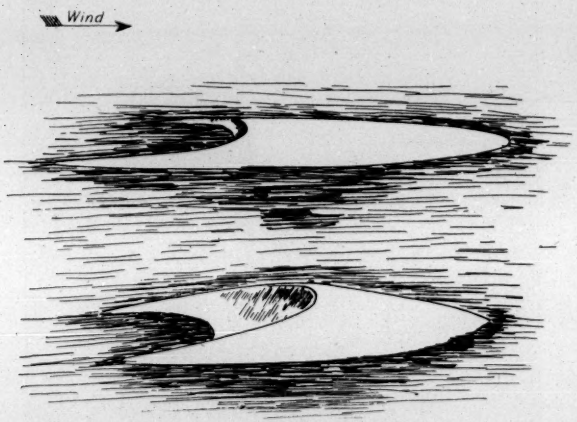
Snow "mushroom." (Taken from Cornish.)

It has always been a matter of much comment that the shapes of many fishes conform very closely to the stream-line contour. According to the view put forth herein any moving body of any shape immersed into a fluid is subjected to forces of restraint that are least when the body has a shape whose contour lines are

stream-lines. Any deformable body will assume that shape. The living organism then has assumed the shape that any deformable inanimate body would. Other familiar examples are found in the case of birds, where not only the shape of the body, but the thickening of the front portion of the wing and the lay of the feathers are conspicuous from this standpoint.

It is perhaps somewhat trivial to point out that the general shapes of many trees and plants is such that as they bend to the wind they present an approximate stream-line contour. It would

FIG. 20.

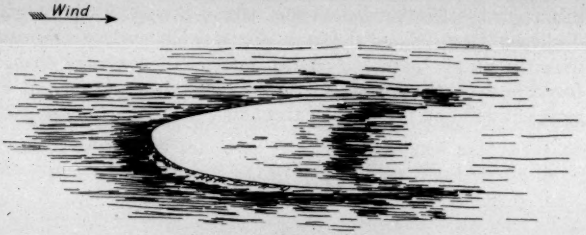


Barchans of snow and sand. (Redrawn from Cornish.)

seem that shapes of seeds and fruit that are dispersed by winds, or by forceful ejections, should present illustrations. I have found none but the proper data are very frequently not on record to enable a decision to be made.[13] Still other illustrations in the realm of biology we should expect to find among the insects, as the butterflies. The features which we are seeking should be enhanced among those species which are quick of flight, strong of wing, and which have relatively heavy bodies. In the so-called hawkmoths (*Spingidæ*) we do find most striking verification

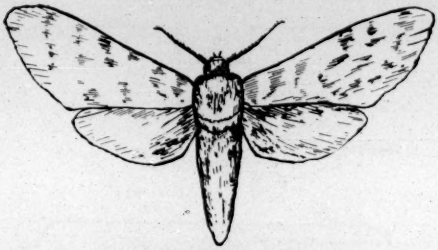
[13] I have Dr. Annie May Hurd to thank for assistance in references in this connection.

FIG. 21.



Erosion form analogous to a barchan. (Redrawn from Cornish.)

FIG. 22.



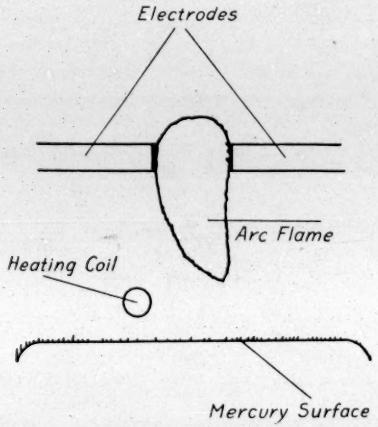
Hawkmoth (*Spingidæ*) having a relatively heavy body of stream-line contour and small but powerful wings.

FIG. 23.

Shape of the flame of the high intensity searchlight arc.

of our principle. I have observed this species rather closely in this regard. The hawkmoth has rather elongated wings, with thickened front edges; the area of the wing surface compared with the weight of the body being appreciably smaller than is found in the case of most lepidoptera. It is, however, one of the most powerful on the wing of them all.[14] The form of its body

FIG. 24.



Shape of the tungsten arc flame.

is certainly most strikingly adapted, as shown in Fig. 22.

The assumption of the stream-line shape by organisms where proper conditions exist may be looked upon in the same light as the adjustment of organisms under a field of force as that of gravity (geotropism), or the orientation, as of leaves, under conditions of directed illumination (heliotropism, phototropism). The forces are constantly present. It would be interesting and be of good evidence to find in case of birds and fish, that there is an adjustment for different conditions of living. For example, the stream-line contour should be different or different speeds of flight and of different relative speeds of current and fish.

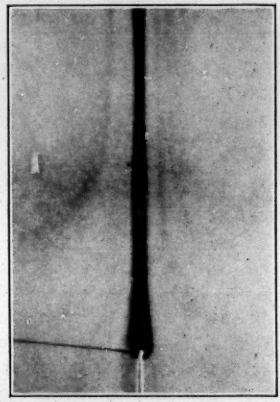
[14] I have seen specimens of it that, while imprisoned in a hat covered with a plate of glass, sustained themselves almost motionless at the centre of the hat for an appreciable length of time.

The phenomenon appears to be of sufficient importance to be specifically designated as the other similar phenomena of heliotropism, geotropism, galvanotropism. For this designation I venture to suggest the term, *rheotropism* (stream adjustment).

e. ILLUSTRATIONS FROM PHYSICS.

The shape of stagnant spots in streaming air has already been referred to. Other instances very closely related to this are the shape of a candle flame, and the shape of the flame or arc in the

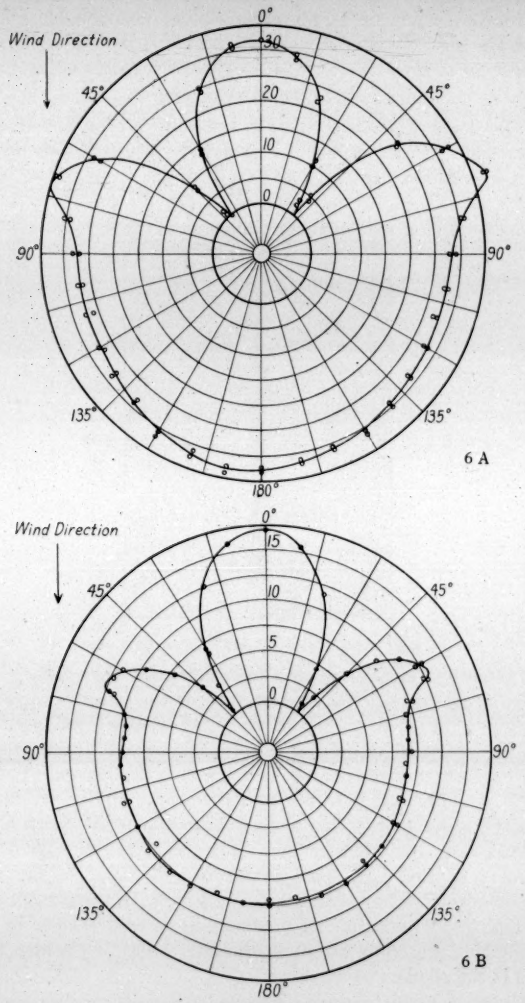
FIG. 25.



Shape of the dark space formed around a heated wire.

high intensity searchlamps so extensively used for military purposes during the late war. A picture of the latter is shown in Fig. 23. The shape of the flame in the mercury tungsten arc, Fig. 24, is likewise interesting in this connection. The streaming in these cases consists merely in the convection currents, which in the instances just cited are rather violent. In Fig. 25, where the dark space that forms around heated wire in dusty air is shown, convection is less violent. Yet even here the shape of the boundary of the dark space in the neighborhood of the wire may be noted. The temperature of the wire in the present case was that of dull-redness. The streaming is made visible by the introduction of ammonia hydrochloride. The horizontal dark space is the shadow of the wire.

FIG. 26.



Distribution of pressures around cylinders in streaming air. (Taken from *Bul. B. S.*, 16, p. 489, 1920.)

1. EVIDENCE FROM THE DISTRIBUTION OF PRESSURE ON A BODY MOVING IN A FLUID.

Dryden [15] has recently reported some data on the distribution of pressure over the surface of a cylinder placed in the wind tunnel. His results qualitatively bear out in a remarkable way the theme of this article. The curves of Fig. 26 are taken from Dryden's paper. The pressure is at a maximum immediately in front of the cylinder, decreasing to either side for an angular distance of about 38°, where there is no change from the static pressure found in the wind tunnel in which the experiments were made.

The pressure drops below the static pressure from 38° to 180°. If this cylinder had been readily deformable its shape should certainly be expected to be altered in a manner already indicated. It is interesting to interpret in the light of these data the shape of icicles which was noted above. If the icicle be a cylinder any film of water present will be pressed away from the front to the rear. The exact shape that the film will take on will be determined by the equilibrium between the wind forces and those of surface tension and adhesion. Icicles are very frequently formed on the eves of dwellings by the trickling of the water over existing forms. Such shaping of the icicle, however, may be expected to take place under conditions where the wind can act merely as corroding agent. It may be pointed out further that the phenomenon of regelation may lend itself as a factor in the reshaping of the icicle when the temperature is 0° C.—the ice melting at points of highest pressure and recongealing at points of lower pressure.

If more data such as that given by Dryden were at hand, showing how the pressure changes with change of shape, one might formulate the relationship between the velocity, size, and resulting shape of the body. However, the principle of stream-line assumption suggests a means of determining the stream-line shape for any condition of streaming. Such a method would be of importance in determining the proper entrance and the proper run of boats, torpedoes and shells. Likewise, a method of recording the relative distribution of pressure over an immersed body is suggested—the body being deformable as a soap bubble, with known forces of restitution.

[15] *Bul. Bur. of Std.*, 16, p. 489, 1920; or *B. S. Sci. Pap. No. 394*, Air Forces on Circular Cylinders. References to earlier data of the same kind are given in Dryden's paper.

No theoretical considerations seem to have been made to predict the distribution of pressure, such as is given in Fig. 26, for a cylinder. Yet it should be possible to make some prediction for simple cases. If it is assumed that the stream of air consists of spheres impinging upon the cylinder, and having a certain average momentum then the pressure at any element of the cylinder will be determined by the momentum imparted to or taken from the spheres in the immediate neighborhood of the cylinder normal to its surface.

It might be expected then that the pressure will be a maximum immediately in front of the cylinder decreasing to either side, first, on account of obliquity between the elements of surface and direction of the stream; second, on account of the fact that the spheres which have already lost some momentum must be swept along with the stream

The general outline of this paper and the experiments on the shape of the mercury drop were made while I was associated with the National Bureau of Standards. More extended experiments on the shape of drops was contemplated there. I recall with much appreciation the friendly criticism of Dr. L. J. Briggs and Dr. H. L. Dryden.

SUMMARY.

Evidence has been assembled to show that any deformable body moving in a fluid or immersed in a moving fluid will assume a shape that is as nearly as is possible "stream-line."

Illustrations of this principle of "stream-line" assumption are taken from geology, meteorology, physics, and biology.

The adjustment of organisms in accordance with this principle is very similar to the adjustment by organisms to directed light (heliotropism) and to gravity (geotropism). Accordingly a term is suggested for defining this adjustment, *viz.*, rheotropism.

CLEVELAND, OHIO,
August 12, 1921.

Surface Tension of Water and of Benzene.—SAMUEL SUGDEN (*Jour. Chem. Soc.*, 1921, cxix, 1483–1492) has re-determined the surface tension of these liquids against air at a temperature of 20.0° C., by measurement of their rise in capillary tubes. The surface tension of water was found to be 72.70 dynes per cm., that of benzene 28.85 dynes per cm.

J. S. H.

DATA PERTAINING TO VISUAL DISCRIMINATION AND DESIRED ILLUMINATION INTENSITIES.*

BY

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Laboratory of Applied Science, Nela Research Laboratories.

NOT many years ago the appraisal of the results of a lighting installation was based largely upon the distribution of illumination intensity upon a so-called horizontal working plane. However, during the past few years the science of illumination has progressed considerably through a much broadened viewpoint, resulting in a more extensive analysis of lighting. Gradually other factors were studied, such as contrasts, illumination intensities on various planes, and the distribution, color and spectral character of light. More recently attention has been directed to various psychophysiological factors such as speed of visual discrimination, the influence of color of light, and the effects of higher intensities of illumination. However, only a beginning has been made in investigating the unknown factors and even many apparently commonplace questions are still unanswered. In the present paper brief discussions are presented of the results of investigations of two phases of visual discrimination, both of which involve intensity of illumination.

In the first investigation the relation of intensity of illumination to the speed of visual discrimination was studied. One of the aims of the work was to develop a method of test which would provide a measure of the influence of intensity of illumination upon this factor without requiring a summary of data; in other words, to devise a " direct-reading " apparatus. A number of test-objects were used such as lines of various kinds, letters and other characters, and different styles of reading matter. The contrast in the test-objects was chiefly that resulting from black ink on white paper, but later this contrast was reduced in the case of the most successful reading test-object (reading matter in Old English type) by decreasing the reflection-factor of the paper. A large number of observers was used.

*Communicated by Mr. Luckiesh.

In the second investigation the aims were chiefly to determine the intensities of illumination chosen by various observers for reading amid an ordinary environment and to compare in this respect illuminants differing widely in spectral character. The test-object was ordinary reading matter having a contrast resulting from black ink upon white paper, and also from black ink on a dark gray paper. The two light-sources were respectively a gas-filled tungsten lamp and a mercury (glass tube) arc. Among other things, the influence of the maximum available intensity of illumination upon the chosen intensity was studied. A large number of observers was employed.

SPEED OF VISUAL DISCRIMINATION AS A FUNCTION OF INTENSITY OF ILLUMINATION.

Many data pertaining to the psychophysiology of vision show that most of the powers of the eye increase with increase in intensity of illumination within certain limits. The recent experiments conducted by Ferree and Rand [1] indicate that certain specific functions, such as speed of discrimination, ability to sustain clear seeing, etc., increase rapidly at the general level of illumination commonly encountered in interior lighting.

In the present investigation an attempt has been made to determine the combined effect of the increase with intensity of illumination of such functions as speed of discrimination and ability to sustain clear seeing, as evidenced by increased efficiency in certain lines of activity in which these visual functions play an important part, with the hope of showing the influence of intensity of illumination upon production in which visual discrimination is involved. The tests were suggested by various processes found in the industries.

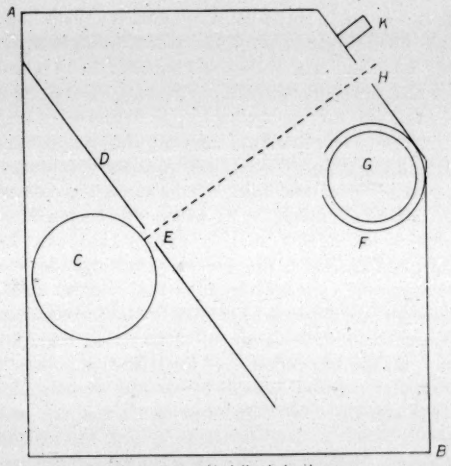
Eventually, after using various test-objects, the central idea was to have an observer read aloud from a column of reading matter attached to a revolving drum, the speed of which he adjusted to correspond to his highest rate of reading under a given intensity of illumination. A screen with a horizontal slot at the centre, placed in front of the drum, concealed from view everything excepting three or four lines of the reading matter, so that when the speed of the drum and the observer's highest rate of reading did not agree, his reading would be intercepted by one

[1] *Trans. I. E. S.*, xv, 1920, p. 769.

or the other edge of the slot. Measuring the speed of the drum when properly adjusted gave a means of ascertaining the rate of reading at different intensities of illumination. Reading aloud required continuous mental and physical activity on the part of the observer, who need not be in any degree accustomed to scientific observation in order to be a satisfactory observer.

Fig. 1 shows the general arrangement of the apparatus. The apparatus was contained in a box or booth, AB, 23 inches high, 21

FIG. 1.



Apparatus for speed of discrimination tests.

inches deep and 18 inches wide as shown in side view. A drum, C, 6½ inches in diameter, was driven through a suitable reduction gear by means of a small motor. The speed of the motor was varied by a sliding rheostat conveniently placed on the front of the booth, and was indicated by the " miles per hour " scale of an automobile speedmotor which was attached to the motor by a section of flexible cable.

The observer viewed the drum through a slot, H, near the top of the box which was provided with head-rest, K, and shields at the sides to eliminate extraneous light. The distance, EH,

from the drum to the observer's eyes was sixteen inches. The drum and mechanism were concealed from view by a screen, D, placed in a plane tangent to the surface of the drum and perpendicular to the line of sight.

The screen had at the centre a horizontal slot, E, ¼ inch wide and 3 inches long, exposing a portion of the test-object on the drum.

The source of light was placed so as to illuminate the central portion of the screen uniformly, and to avoid glare. In order to maintain a constant quality of illumination the use of a rheostat to secure variations in intensity was avoided. The illuminant, a 75-watt gas-filled tungsten lamp, was placed in a glass cylinder, G, 8½ inches long and 4¼ inches in diameter, which had a non-selective white diffusing surface. The axis of this cylinder was parallel to the axis of the drum, C. A series of wedge-shaped strips of tinfoil, of length a little less than the circumference, were pasted around the cylinder with their bases on a common element of the surface. The remainder of the circumference was completely covered with tinfoil. The whole was enclosed in a sheet-metal cylinder, F, having in it a longitudinal slot about 1½ inches wide, and as long as the diffusing glass cylinder.

By rotating the cylinder about its longitudinal axis the area of luminous surface exposed by the slot in the sheet-metal cylinder would be varied, with a resultant variation in the intensity of illumination on the test-object. A knob with a pointer and dial was provided at the side of the booth for rotating G and for reading the intensity of illumination after the calibration was made.

This type of variable source gave a uniform illumination over a sufficiently large area of the screen; it varied the intensity of illumination of the test-object from zero to 25 foot-candles; and it could be accurately calibrated and maintained constant.

Several kinds of test-objects were devised and used in the preliminary tests. Among them were sets of parallel lines, geometrical figures, isolated letters and numbers, which were allowed to flit rapidly past the slot in the screen. The observer was required to reduce the speed until he was just able to distinguish the characters. Here the motion of the test-object affects its discriminability, as is shown by the fact that if the characters on the drum consist of circles with a break in the circumference,

the break is more easily seen when at the top or bottom of the circle than when at the side. In the reading test, the motion of the drum is so slow that this effect is not present.

It is true that the speed of reading reaches a limiting value depending on factors other than speed of discrimination, and in order to make it a criterion of the latter, it is necessary to keep below this limiting value. Reading matter in ordinary type was found to be unsuitable as a test-object, because the limiting speed is reached very soon after the visibility of the test-object becomes great enough to discriminate it at all, even if very small type is employed. It appears that the form or outline of the word taken as a whole plays an important part in the recognition of familiar words printed in ordinary type. Old English type has the advantage of eliminating these characteristic outlines. The individual letters also contain more detail than those of ordinary type so that close discrimination is necessary to read it.

The test-object finally selected had the following qualifications: The type was exceptionally clear-cut and uniform Old English. The material was in the form of narrow columns without paragraph indentations. It contained no uncommon words and the subject-matter was such as would not require unusual thought or concentration on the part of the reader.

Photographic copies of the original were made, the size of the type being reduced to about that of 8-point ordinary type. The columns were 1¼ inches wide and were pasted end to end on the drum so as to form continuous columns of reading matter around it. Two such columns provided enough new material for a complete test.

In many industrial processes where speed of discrimination is an important factor, the extreme contrast of black on white is seldom encountered. Probably in many cases the greatest diffuse reflection-factor would not exceed 20 per cent. It accordingly seemed that the results of the test could be made to apply more closely to actual processes by reducing the contrast of the test-object.

Two series of tests were run, in one of which the coefficient of reflection of the background of the test-object was 80 per cent. (white) and in the other 22½ per cent. (a neutral gray produced by fogging the photographic prints). The screen, D, which filled practically the entire visual field, was matched to the back-

ground of the test-object in each case. For 80 per cent. reflection it was covered with white blotting paper, and for 22½ reflection with a neutral gray paper with a matte surface.

Observations were taken at four different intensities of illumination. For the low-contrast test-object these were 3.12, 6.25, 12.5 and 25 foot-candles at the screen, D, and for the high-contrast test-object 0.39, 1.56, 6.25, and 25 foot-candles.

There are other factors besides intensity of illumination which might, if not eliminated, have an influence on the speed settings of an observer. Inferring that he was expected to increase his speed at the higher intensities, he might do so voluntarily if there were any circumstance which furnished him a means of comparing his different settings. Accordingly the speedometer was located so it could not be seen while making observations. The noise of the mechanism which might possibly form a criterion was reduced to a minimum through the use of rubber pads. The possibility of basing the settings on the position of the rheostat slider was eliminated in that a second rheostat, in series with that operated by the observer, was provided at the side of the box, and after each setting its resistance was altered in a haphazard manner.

Two other important considerations in determining the procedure are the state of retinal adaptation of the observer and the effect of increasing familiarity with the Old English type. A prolonged period of adaptation is of overwhelming importance when the level of illumination is near the lower limit of visibility or when the eye is successively subjected to widely different intensities.

In the present case the general level of intensity was that of a well-lighted interior, which was that to which the observers were adapted before commencing the test, while the intervals between the different levels of intensity employed in the test were very small as compared with the total range of intensities under which the eye is called upon to function. A period of three or four minutes was allowed for adaptation at each intensity. The observer was simply permitted to read until his reading became constant and the speed of the drum had been correspondingly adjusted.

In order to minimize the effect of increasing familiarity with the type, two settings at each intensity of illumination were taken. The test was begun at the lowest intensity and the illumination was increased by steps to the maximum value, and then brought

back to the starting point over the same route. An actual determination of the rate of reading as a function of the time at constant intensity of illumination (12.5 foot-candles) for several observers who were unfamiliar with the test-object showed very little increase after the first minute or two, after which time the relation was approximately linear, so that the above procedure would practically eliminate this factor from the results. The speedometer was carefully calibrated in terms of revolutions per minute of the drum.

Data from 49 different observers were obtained with the low-contrast test-object, and 37 observers with high-contrast object. Nearly all the observers in the second series had participated in the first series, but this could have little effect on the results other than to further minimize the effects of the "learning curve."

In most cases the speed settings on the return trip were higher than on the initial trip, as might be expected from the above considerations. The results showed wide differences between individuals both as to absolute speed of reading and relative increase of speed. Ferree and Rand found that slight defects of refraction of the eye cause speed of discrimination to increase more rapidly with intensity of illumination, a fact which in part accounts for the differences between individuals. To show the sort of agreement obtained between individual observations, the results for two observers, which may be regarded as typical, are included.

TABLE I.

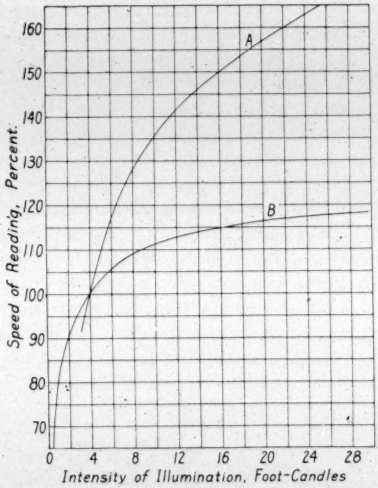
Observer	Intensity of Illumination Foot-candles	Speed Settings, Illumination Increasing (Read down)	Illumination Decreasing (Read up)	Average
F. W. P.	3.12	17.5	26	22
	6.25	26.5	30	28
	12.5	33.5	32.5	33
	25	38	40.5	39
E. P.	3.12	11	12	11.5
	6.25	13	15.5	14.5
	12.5	17.5	19	18.5
	25	22	21.5	22

In order to obtain an average of the results the speed at 6.25 foot-candles for low contrast and 1.56 foot-candles for high contrast for each individual was set equal to unity.

Fig. 2 represents the mean results of all observations, plotted against intensity of illumination at the test-object. For both the low-contrast and high-contrast tests the speed at 4 foot-candles is taken as 100 per cent.

Fig. 3 shows the results plotted against the brightness in millilamberts of the background of the test-object. Here the speed at a brightness of 1.5 millilamberts is taken as 100 per cent. in

FIG. 2.



Effect of intensity of illumination on speed of reading. A, low-contrast test-object. B, high-contrast test-object.

both cases. It is seen that when speed is plotted against brightness of background, the test in which the background had the low reflection-factor still shows a greater relative increase. This was to be expected from the fact that the contrast was actually lower in this case, since the type face reflection-factor was approximately equal for the two test-objects.

The results show an average increase in speed of reading of 64 per cent. for the low-contrast test-object when the intensity of illumination was increased from 4 to 25 foot-candles and indicate

that the maximum speed had not been fully attained at the higher intensity. For the high-contrast test-object the average increase was 17 per cent. between the same limits and the curve approaches horizontal at 25 foot-candles.

TABLE II.

Data for Figure 2.

Intensity (Foot-candles)	Speed of Reading, per cent.
High-contrast test-object	
0.39	65.1
1.56	87.1
6.25	106.3
25	117.6
Low-contrast test-object	
3.12	91.8
6.25	119.0
12.5	142.8
25	164.3

TABLE III.

Data for Figure 3.

Brightness of Background	Speed of Reading, per cent.
Low-contrast test-object	
0.75 millilambert	77
1.50	100
3.0	120
6.0	138
High-contrast test-object	
0.33	73
1.33	97.5
5.33	119
21.32	131.5

ILLUMINATION INTENSITIES DESIRED FOR CONTINUOUS READING UNDER TWO ILLUMINANTS DIFFERING WIDELY IN SPECTRAL CHARACTER.

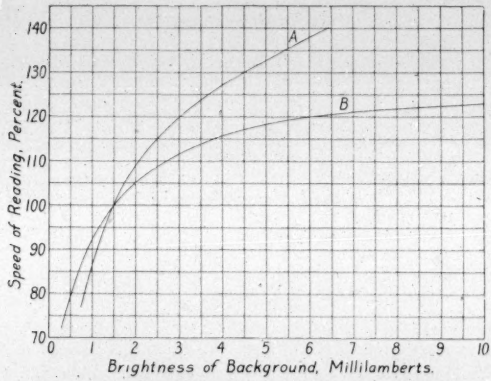
Some years ago Bell,[2] Luckiesh,[3] and others studied the effect of the spectral character of light upon visual acuity. The results of their researches showed that, owing to the chromatic aberration of the eye, visual acuity increased in general as the spectral character of the illuminant approached monochromatism. However, this result was obtained for details of the smallest size visible; in other words, at the limit of visibility.

[2] *Electrical World*, May 11, 1911; Sept. 9, 1911.

[3] *Electrical World*, Aug. 19, 1911; Nov. 18, 1911; 62, 1913, p. 1150; *Trans. I. E. S.*, 7, 1912, p. 135.

It is well known that visual acuity does not increase as rapidly with increase of intensity of illumination for the level of intensities ordinarily used for reading, etc., as for lower levels of illumination. Furthermore, there is a general tendency to overlook the fact that seldom are the eyes called upon to distinguish objects of a size approaching the limit of visibility. Even in reading it has been shown by various investigators that the eyes are not concerned with visual acuity in the sense of limiting values. For example, type is commonly much larger than the smallest which may be distinguished under usual conditions, and any advantage

FIG. 3.



Relation between brightness of background and speed of reading.

in "revealing power" possessed by one illuminant over another for details of the smallest size which are visible decreases as either the size of the details or the intensity of illumination increases.

There are also interesting aspects which pertain to the eye and to its performance when engaged in seeing. It is well known that fine details can be distinguished only when looking directly at them. In other words, visual acuity is high for the central portion of the retina (the fovea), but it rapidly diminishes for areas beyond this region. In reading, it has been well established that the eyes do not scan each letter or even each word. The point of fixation progresses by a series of "jumps" along a line of type and experiments show that the number of "jumps" varies usually

from about two to seven for ordinary lines, say four inches in length. This is proof that visual acuity cannot be exercised in the ordinary sense in reading under usual conditions.

The chromatic aberration of the eye is easily demonstrated by viewing a line spectrum focused upon a ground glass. Ordinarily the normal eyes are focused upon reading matter at a distance of about twelve to fourteen inches. A yellow or green line in the spectrum can be easily and distinctly seen at about this distance; on the other hand, the blue and violet lines in the mercury spectrum are easily focused by a normal eye at a distance of four or five inches, and cannot be focused at the normal distance. The phenomenon is not so marked at the red end of the spectrum because red spectral lines would normally be focused at greater distances than yellow lines.

These are interesting facts but after all there are so many subtle and intangible influences in lighting that the final test is that of actual practice. For this reason it appeared of interest to conduct some practical tests, under good conditions of distribution of light in an ordinary room (with light ceilings, buff walls and light oak finish and furniture), using observers generally unacquainted with such facts as are discussed in the foregoing, to determine the relative importance of some of these factors in actual practice.

The apparatus and method of procedure were changed two or three times during the tests, whenever it appeared that conditions could be improved, or tests under different conditions were desired. The specific changes made will be described later. Observers were obtained from the Research Laboratories and the Engineering Department, availability being the only requirement.

In the early part of the work gas-filled tungsten lamps (emitting light of an extended spectral character) were arranged in a white-lined box with a shutter in front. A similar apparatus was made up with a mercury-arc lamp (glass tube), emitting light approaching monochromatism. The observer was seated at a table in an office and read a book on the table in front of him. The illumination, received over the left shoulder from the lighting units placed above and behind him, could be varied at will by the observer by means of strings attached to the shutters. Especial care was taken to avoid glare by specular reflection. When the illumination on the reading material was varied the illumination

TABLE IV.

Summary of Results af Reading Tests.

Test No.	No. of Observers	Max. Illum. Obtainable.	Foot-candles Chosen by Group				Ratio of Illumination Intensities Mercury Arc to Gas-filled Tungsten				Prob. Error of Mean of Individual Ratios
			Mercury Arc		Gas-filled Tungsten						
			Mean Value	Median Value	Mean Value	Median Value	Ratio of Means	Ratio of Medians	Mean of Individual Ratios	Median of Individual Ratios.	
1	*22	10 f. c.	6.3	6.5	5.3	5.6	1.19	1.16	1.20	1.20	±.036
2	55	30	14.2	13.5	12.7	12.3	1.12	1.10	1.17_5	1.14	.019
2a	24	30	11.8	10.5	10.6	9.6	1.11_5	1.09_5	1.16	1.15	.012
3	26	30	18.0	17.8	17.4	16.0	1.03_5	1.11	1.06_5	1.07	.014
4	24	45	16.9	16.2	16.1	14.7	1.05	1.10	1.10	1.04	.012

Test No. READING MATERIAL

1. *Trans. Illum. Eng. Soc.*, also "Life of Edison"
2. *Scientific American Monthly*
2a. " " "
3. *Saturday Evening Post* (dyed)
4. *Scientific American Monthly*

* Fifteen of these observers repeated, and their second observations have been given the same weight as different observers. The same observers were used in Tests 2a and 4. Results of 2a are included in 2.

of the walls and other objects in view varied in like manner. The maximum illumination of the reading material obtainable from each source was approximately ten foot-candles.

The reading material, chosen to be of the most probable interest to all observers, was a copy of the *Transactions of the Illuminating Engineering Society,* and another book with matte paper, "The Life of Edison." Each observer, when seated at the table, was told to adjust the illumination intensity to the value which he would consider necessary for comfortable reading, without eye-strain, for several hours. The idea which we attempted to convey was that the observer should not try to use the minimum possible illumination, but neither should he be wasteful of the light. The observer readjusted the illumination at the end of every five minutes for a period of one-half hour. Later in the day he repeated the observation, using the other illuminant. In working up results of the test the first setting by each observer was disregarded, as it was felt that the first five minutes were necessary for adaptation. Twenty-two observers took part in this test, and fifteen of these repeated their observations. In the test results given in Table IV the second observations by these fifteen have been treated as observations by other individuals.

Profiting by the experience gained in this test, some changes in conditions and procedure were made. Illuminants of the two types were suspended about five feet above a table placed in the corner of a large room. A fifty-inch glass mercury tube with reflector was used.

For about three-fourths of the observers in test No. 2 (Table I) the tungsten source was a large gas-filled tungsten lamp in a housing behind an opal glass disk about ten inches in diameter, screened by a cornu diaphragm. In order to make the two sources more nearly alike as regards diffusion a new tungsten unit was later prepared. Ten gas-filled tungsten lamps were placed in a white-painted box 15 x 36 x 9½ inches, the opening being covered with opal glass. Roller curtains were arranged with both illuminants to screen them to the desired amount, beginning at both ends of each. The position of the table was such as to avoid glare by specular reflection. The reading material used was a copy of the *Scientific American Monthly.* In order that all observers should receive the same instructions, written instructions as follows were given to each:

INSTRUCTIONS FOR READING TEST.

"The object of this test is to measure the intensity of illumination most satisfactory for continued use in reading. This does not mean an intensity with which it is just possible to get along, but rather one which seems comfortable and desirable.

"In commencing the test the observer will read for a period of four minutes under a selected intensity of illumination. He will then be instructed to adjust the light to suit his own taste, and in doing so will bear in mind the above-mentioned objective. At the end of each four-minute period the observer will be instructed to readjust the light."

The selected intensity referred to was in most cases four foot-candles. Observations with each source continued for sixteen minutes, one following the other immediately. Alternate observers were started with the tungsten source, others with the mercury-vapor. For the major part of this test the maximum intensity obtainable with each source was thirty foot-candles. Later a further test was made under the same conditions, with a maximum intensity of forty-five foot-candles with each source.

In order to determine the effect of reduced contrast in the reading material a copy of the *Saturday Evening Post* was dyed a dark gray, its reflecting power being almost exactly half that of the *Scientific American Monthly* which had previously been used. The maximum intensity obtainable in this test was thirty foot-candles.

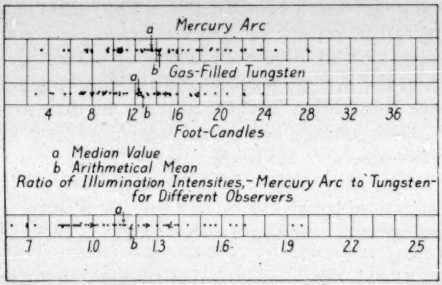
In most cases photometric observations were made by two or three observers. Each source had a scale indicating the shutter position, and calibration curves were determined. Constancy of illumination was checked frequently. The mercury arc was found to be variable in intensity when burned on constant voltage, but fairly steady when burned at constant current on a 230-volt circuit with extra resistance.

In tests of this nature results by different observers are likely to vary over a wide range. The result for a group of observers may be expressed in several ways, two of which are the arithmetical average and the median value, *i.e.*, the value for which there are equal numbers of observers above and below. The median has the advantage of not giving undue weight to observations which are far from the average. The results of these tests will be given in both ways, though in a few cases values by

one or two observers will be omitted from the summary because they deviate greatly from the average (this is justified by Chauvenet's criterion). Observations by three observers in one of the tests are omitted because they selected the maximum intensity available with one or both illuminants. This raises the question whether they properly understood the criterion of observation, or whether or not they might have selected a higher intensity had it been obtainable. In beginning this second test it was thought that thirty foot-candles should be ample for any observer.

A summary of results of the various tests is given in Table IV. A plot of the results of Test No. 2, which is typical of all the tests,

FIG. 4.


a Mercury Arc
b Gas-Filled Tungsten
4 8 12 16 20 24 28 32 36
Foot-Candles
a Median Value
b Arithmetical Mean
Ratio of Illumination Intensities,-Mercury Arc to Tungsten-
for Different Observers
7 10 13 16 19 22 25

Average illumination intensities chosen by different observers for continuous reading under mercury arc and gas-filled tungsten lamps. (Test No. 2.)

is given in Fig. 4. The probable error of the mean of the individual ratios in each test, given in the last column of Table IV, has been computed by means of the laws of probability.

In Table V are given data regarding the reading materials used in the tests. The "apparent diffuse reflection factor" used in the table may be defined as the ratio of the brightness of the unprinted portion of the material, at the angle of view (about 30° from the normal), to the brightness of a perfect diffuser, having a diffuse reflection-factor of 100 per cent., receiving the same illumination. In order to convert this factor into brightness in millilamberts for an illumination of one foot-candle, it should be multiplied by 1.076.

TABLE V.

Data on Reading Materials Used in Tests.

Reading Material	Apparent Diffuse Reflection-Factor	Size of type	Lines per Inch
Trans. I. E. S.	84 per cent.	11 point	6
"Life of Edison"	89.5 per cent	12 point	5.6
Sci. Am. Monthly	80.5	9 point	7.2
Sat. Eve. Post (dyed)	*41	9 point	9
Sat. Eve. Post (dyed)	*42.5 for mercury-vapor light.		

*The dye used in dyeing the *Saturday Evening Post* was somewhat bluish, which accounts for the higher reflection-factor obtained for illumination by the mercury vapor lamp. If this correction was applied to the results in Table 4, all ratios for test No. 3 would be raised about 4%.

The visual acuity tests by Bell and by Luckiesh, referred to in the first part of this paper, indicated that at illuminations of the order of one or two foot-candles about 75 to 95 per cent. more light from tungsten lamps than from mercury arcs was required for equal revealing power of small details. The results of the tests reported here appear to show that visual acuity at low illuminations is not a proper measure of the usefulness of a light for clerical work. In fact, the average observer desired from five to twenty per cent. more illumination from the mercury arc than from tungsten lamps, and in no case did the average ratio for a group of twenty or more observers fall below 1.04.

The actual intensity desired by the average observer is not so definitely determined but appears to be from ten to seventeen foot-candles for ordinary reading matter when a high intensity is available.

Attention is directed to the fact that in test No. 4, which was identical in nature with No. 2a, except that the maximum intensity obtainable was 50 per cent. greater, the average intensities chosen were very closely 50 per cent. greater than in test No. 2a. However, the ratio of mercury to tungsten illumination was not greatly changed. It would be of interest to determine the maximum intensity which the average observer would choose if the available intensity was unlimited, but unfortunately lack of apparatus to obtain still higher intensities precluded the performance of such a test.

The authors desire to express their appreciation to all the observers who participated in the tests, and especially to the Engineering Department, which furnished approximately half of the observers.

CLEVELAND, OHIO,
JULY 21, 1921.

PHYSICS A HUNDRED YEARS AGO.*

BY

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A CENTURY ago science had recently lost three eminent men who had notably advanced our knowledge of electricity, dynamics and heat, Cavendish (1731–1815), Rumford (1753–1814), Watt (1736–1819).

The steam engine had appeared and was used for pumping mines, for locomotives and for the propulsion of ships; the notable discovery had been made, to quote the contemporary words of John Herschel, "A man's daily labour is about four pounds of coal." "Two pounds of coal would raise a strong man from the valley of Chamounix to the top of Mt. Blanc." "You can raise seventy million pounds weight a foot high by a bushel of coals." [1]

There had just begun that industrial revolution due to the use of coal and iron, which, for better or worse, has in a century transformed the world.

Every age regards its progress with a wholesome and justifiable pride. The achievements of preceding generations are dimmed in lustre by familiarity. The imagination is too feeble to form an adequate conception of the marvels awaiting discovery, ready to fall like ripe plums into the laps of successors. On the other hand, recent discovery always stands out with a delightful and refreshing vividness.

Now a hundred years ago people were thoroughly pleased with their discoveries, no less than we are to-day. It is sufficient to mention such successive discoveries as the spinning jenny (1768), spinning frame (1769), cotton gin (1792); the discovery of the planet Uranus (1780), the first air balloon (1783), and vaccination (1796).

Thanks to Newton and others, it was a just claim, in 1821, that more scientific progress had been made in the preceding two hundred years than in the whole previous history of mankind.

* Address at the Centenary Reunion of McGill University.

[1] The actual work done by a bushel of coals used in a steam engine was called its *duty*, a useful term.

It is curious to read moreover the lamentations by Thomas Young on the enormous amount of scientific literature and the great variety of publications, which rendered it difficult or impossible to keep abreast with scientific discovery. How seriously has this evil increased during the past hundred years, until we seem doomed to be buried under our own records! And this trouble must continually increase with time.

Mr. James McGill was an enlightened citizen of Montreal with an interest in literary and scientific progress. It requires but a small stretch of the imagination to conceive of our founder sitting under an elm tree on Burnside Farm by the side of that little brook, with its rustic bridge and lovers' walk, which flowed past the spot where the Macdonald Physics Building now stands. The valley of that brook is still visible in the back lane and tennis court. And indeed in spring time, the brook itself revives and floods our basement.

Imagine him seated there and reading the following fictitious letter supposed to have been written about a century ago by a friend of James McGill, an imaginary professor of natural philosophy at the famous University of Glasgow, giving an account of a visit to London and Paris, and describing to our founder what he saw which was new and interesting in the scientific field. It is a matter of regret to me that I cannot read this letter to you in the good Scots tongue.

From Professor Robin Angus,
The University of Glasgow,
(Undated).

To Mr. James McGill,
of Montreal.

Dear Mr. McGill,

I am now fortunate in writing to you to give my promised account of a long projected visit to London and to Paris, and my description of the progress of recent discovery in natural philosophy.

I left Glasgow on the first of June and the roads were in good condition so that we made a swift and agreeable journey. One day indeed we traveled 59 miles in 11¾ hours, including time for baits!

On my arrival at London I quickly went to the Royal Institution and called on Dr. Thomas Young. I was fortunate enough to hear one of the 93 lectures which he is giving on natural philosophy. These lectures are shortly to be published as a book, a copy of which I will send you. His lectures were well illustrated by skilful experiments.

You are aware that Sir Isaac Newton suggested that light consisted of little bodies or corpuscles shot from the source of light traveling "with an eel-like motion" along straight lines. Now Dr. T. Young will have none of this theory, but he agrees with Huyghens that light travels with wave motion in some subtle and all pervading medium which is called æther. Huyghens thought that light consisted of waves with a motion of the æther to and fro in the direction in which light traveled, but Doctor Young points out, as did Newton, that light may be "one-sided" or polarized, so that it is essential to believe that the vibrations are transverse or perpendicular to that direction in which light moves. As indeed the French philosophers have very clearly proved.

Doctor Young has a large trough with a glass base, filled with water, illuminated beneath; and with a large mirror he projects upon a white screen the waves which are made upon the water by one or more pointers fastened to vibrating rods. In this manner he illustrates very clearly what is called the interference of light, well enough known to Newton, but a stumbling block to his corpusular theory.

At the Royal Institution I met also with Sir Humphrey Davy, who has saved countless lives of miners by his safety lamp, where the flame is surrounded by fine wire-screen, preventing premature explosion.

The great Corsican ogre, Napoleon, scourge of the world, is newly dead. Yet in fairness it must be stated that he proved a good friend to science. In the midst of the war between England and France he gave, in spite of strong opposition, a great scientific prize to an Englishman, Davy, for his discovery of potassium and of sodium by electric separation. He caused a galaxy of scientific men to gather at Paris, and encouraged them in their work by every means at his disposal. Napoleon was a man who certainly knew that in science, too, "As a man sows, so shall he reap."

I met at the Royal Institution a young assistant of Davy's named Faraday who was full of insight and enthusiasm so that

he promises to go far. He was greatly interested in electrical experiments.

You are familiar with electrical machines and Leyden jars, lightning rods and Franklin's experiment with the kite, and how he obtained electricity from the clouds. All these are well described in a little book by Doctor Priestley [2] which I sent you last year. But, as the Hon. Mr. Cavendish wrote, "It must be confessed that the whole science of electricity is yet in a very imperfect state"; or to quote my friend Doctor Young (p. 507), "The phenomena of electricity are as amusing and popular in their external form as they are intricate and abstruse in their intimate nature."

Suddenly there has come from Denmark a great burst of light, which we owe to Hans Christian Oersted. This illustrious man was born in 1777, and after passing with honours at school he received *free* residence and a small scholarship awarded to needy students. [3] After a distinguished career at Elers College he received a Cappel Traveling Fellowship which enabled him to visit the leading scientific men in Germany and France to his great benefit as it now proves to ours.

This plan of helping able students to secure a good university education, and to visit other countries in order to appreciate scientific progress, has much to commend it to other countries and to all universities.

Many philosophers have endeavoured to deflect a magnet with electricity, using an electrical machine with open circuit. Now Oersted was lecturing to his advanced students and he discovered, his class being there and then assembled, that with an electric battery and a closed circuit he could cause a current of electricity to deflect a magnet. Not when the wire is perpendicular to the needle, but when parallel. This influence will pass through wood and water and mercury and metal plates, excepting iron, so that the influence of the electric current on a magnetic pole is as it were in circles around the wire. Already Schweigger, at Halle, has invented a measurer of electric current called the Astatic galvanometer, where two equal magnetic needles pointing opposite ways have

[2] "A Familiar Guide to the Study of Electricity," 4th ed., 1786. (J. Johnson, London.)

[3] *Nature*, p. 492, 16 June, 1921.

been deflected by a current passing in a coil of wire round one needle, a most sensitive arrangement.

Davy, using the great battery of 2000 cells of zinc and copper at the Royal Institution, has passed an arc between two carbons giving a most brilliant light. Now this arc he has deflected with a magnet, showing that as a current in a circuit will deflect a magnet so will the magnet deflect the circuit if and when a current passes in it. Here then we have another example of the third law of Newton that "action and reaction are equal and contrary." Nay! Oersted himself hung up by a fine wire a small battery and coil and deflected it with a magnet. Hence we now have a new branch of science, my dear Mr. McGill, which we may call electrodynamics or electromagnetics. The great M. Ampère at Paris has made vast strides in this new subject.

And indeed I must pass over much that I would wish to tell you that I saw and heard in London, and proceed with my visit to Paris, which I reached safely after a troubled crossing over the Channel.

In spite of the recent wars, most cordial relations have speedily returned between scientific men of all countries.

I have met M. Ampère who, stimulated by Oersted's discovery, has extended it and proved that "two parallel and like-directed currents attract each other, while two parallel currents of opposite directions repel each other."

It may be truly said that "the theory and experiment (of electric currents) seem as if they had leaped full-grown and full-armed from the brain of the 'Newton of Electricity.' The theory is perfect in form and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced and which must always remain the fundamental formula of electrodynamics." [4]

But I must pass on, my dear Mr. McGill, to other branches of natural philosophy. I must name the illustrious M. Chladni, whom they call "the Father of Acoustics." Him Napoleon summoned to show his experiments on sound and gave a grant of money towards the publication of his book. Galilei first experimented with dust on vibrating metal plates struck by a chisel, but Chladni made great improvemnts by using lycopodium dust with sand. He separated thus the quiescent from the turbulent regions.

[4] Maxwell.

for as Faraday has explained, the light lycopodium dust is caught in the whirlwinds of air and finally comes to rest below them, while the heavier sand is driven to the nodes. I have been informed that in the recent wars sand has been placed on a drum and the direction of underground mining has been found by the displacement of the sand on the top of the drum set vibrating by the distant blows on the ground of the picks of the enemy. An ingenious application of Chladni's figures!

Most interesting of all are the speculations about light founded on the most ingenious experiments carried out by Fresnel and Arago. They experiment with "one-sided" or polarized light and secure interference between two rays from the same source polarized in the same plane, which cannot be done when the rays are polarized at right angles. This is strong evidence for the wave theory, but a challenge was given that a small round body like a coin should have a bright spot in the center of its shadow from a small bright source of light. In truth, and it should! And the difficult experiment was triumphantly carried out by M. Fresnel!

Beautiful and interesting experiments have also been carried out by M. Malus on the polarization of light, and splendid colour effects have been achieved with the interference of polarized light passing through crystals of mica, gypsum, or quartz.

The simplest interference experiment is to pass light through a slit and hence through two slits close together. On a screen behind you can perceive bright and dark bands alternating which prove that two lights can make darkness, which seems impossible with material things, but is readily explained with waves, for we have all seen, on a lake or pond, crests and troughs of waves cancel one another.

There is great encouragement given to science in these days. Thus the famous Euler received a grant of £20,000 in the last century, and the British Government offered a prize of £20,000 for finding the longitude at sea within thirty miles.

Space has not permitted me to write of Fourier, a great mathematician who has established most fundamental principles of the flow of heat. His work, "Théorie de la Chaleur," has in his own lifetime passed into a classic.

But what shall I say of Laplace, author of "Mécanique Céleste," now seventy years old, comparable only with Newton, who has been honoured by all political parties in the turbulent

periods passed by France in his long life. A man more admired than loved perchance! Laplace has advanced the theory of tides, explained the origin of the sun and planets from a nebula to its present state, and proved that all bodies of the solar system are stable, and may have been so for periods of vast antiquity.

In the spectrum of the sun, Wollaston (1802) and Fraunhofer (1815) have found a very great number of dark lines which await explanation from succeeding generations. Here indeed we have a great mystery!

But I fear, dear sir, that my letter has far outstripped your patience. Your friends in Glasgow and in Scotland learn with pleasure and interest your scheme for founding a College for the Advancement of Learning in Montreal. Judging from what I have seen in Scotland, in England and in France such an institution may bring lasting lustre to your name, and yield priceless fruit throughout succeeding ages.

Believe me, honoured sir,
Your most respectful servant,
ROB. ANGUS.

It must be admitted that historically the above letter will be found wanting, for it purports to be written in 1821, by a "fake" professor of Glasgow University, whereas we all know that our founder died in 1813, eight years before McGill received its charter in 1821. I am assured, however, by my colleague, Prof. Cyrus Macmillan, that otherwise my conception of such a letter is a sound one, and that James McGill was truly interested in science as well as letters. He was himself a student or at least a matriculant of Glasgow University, a fact which explains so much. You will recall that he specially enjoined in his will that there should be a Professor of Natural Philosophy, until such time as there should be three chairs established in mathematics, natural philosophy and astronomy.

To-day McGill has many professors of physics, a subject now taught to all faculties. McGill has also several professors of mathematics, but no astronomer, although he whom we might venerate as our second founder, I name Sir William Macdonald, donated a splendid region on the summit of Westmount for an observatory, the land being still available, although we cannot hope for "good seeing" within the confines of a city yearly growing

blacker with factory and engine smoke, largely preventable and unnecessary.

As for the information conveyed in the fictitious letter, it is gathered mainly from contemporary sources, and the lectures by Dr. Thomas Young, afterward published as a Treatise on Natural Philosophy, are a great mine of information. But a more valuable source is Mrs. Kirstine Meyer's recent essay [8] on the Life of Oersted. For in 1801 Oersted went to Weimar, Berlin, Gottingen and Paris; he saw Ritter's electrical experiments and the very first storage battery, copper plates with damp cardboard between, which retained a charge for some time after it was connected to a battery, capable also of generating a current after being charged.

In 1812 and '13 Oersted again visited Berlin and Paris, and from autumn, 1822, to the summer of 1823 he visited Germany, France and England, although he was full professor of natural philosophy at Copenhagen at the time. I mention this because we see here in the same man the great advantages of three notable institutions or arrangements which I wish to advocate ardently for Canada and elsewhere. For in the case of Oersted we see an able but needy student obtaining free board and residence and a scholarship as well, relieving him of money embarrassments and securing him a sound and liberal education. Secondly, we find him with a traveling fellowship which enabled him to appreciate the work and progress of many scientific centres. Lastly, we find him with a sabbatical year, relieved from the burden of teaching and academic affairs, and given leisure to think and to investigate. The scholarships, the traveling fellowship, the sabbatical year were all fruitful. As a result Oersted founded electrodynamics, for he proved that a coil of wire with a current round it was the equivalent of a magnet.

This fundamental result, developed by Ampère, Faraday, Maxwell, and many other co-workers, is the seed of the fruitful results or harvest which you see around you to-day. I refer to electric motors, lamps, dynamos, generators, electric irons, cookers, bells, toasters, cleaners, and no less to telephones and telegraphs.

We can rest assured that if you give due encouragement and assistance to your quite ablest boys at schools, and to students and professors at universities there are other and greater conquests of

[8] See *Nature*, 16, June, 1921.

science of which we have little or no conception to-day, awaiting discovery and development, and that you must not hesitate to encourage pure research, at unpromising subjects even, rather than endeavor too much to secure industrial research on a commercial basis. The pioneer work is truly of the greater importance though less likely to secure the appreciation of manufacturers, of politicians, of practical men and of the public at large.

Here I must interpose a story. About fifteen years ago, one of my predecessors, Professor John Cox, gave a lecture in this theatre on the passage of electricity through rarefied gases combined with some wonderful experiments, all with the skill and eloquence of which he was and is still a master. Now Sir William Macdonald was present and he remarked afterward, "How beautiful and how useless!" Yet it is the study of those very phenomena which has led to most notable recent developments in radiology, for example the Coolidge tube, in long-distance and guided telephone, in wireless telephony and telegraphy, particularly by the use of the electronic valves.

But Sir William appears to have been himself a convert before his death. As donor to McGill of this Macdonald Physics Building, as founder of the two Macdonald chairs of physics, he was present at a lecture given by Sir Ernest Rutherford on some of his recent work on radioactivity, and after the lecture Sir William stated that "if all the money spent on the endowment of physics at McGill had produced no other result but Rutherford's work on radioactivity alone—the money would have been well spent!" That verdict you will all endorse, with a fervent hope that, although we can scarcely expect ever to rival that remarkable outburst at McGill, of a new branch of physics, we may not merely assist in the training of many thousands of young Canadians in the foundations of science, but also hand on the torch of original research and pioneer investigation in this place.

Oersted in 1822 and '23 was not very enthusiastic about German science. "Schweigger, at Halle, has brains, but is a reed shaken with the wind. His experiments are not of much importance; Kastner, at Erlangen, writes thick volumes compiled with much toil but without all judgment. Yelin, at Munich, makes indifferent experiments and lies much." (Really, really, Yelin, this is too bad!) "But I have found much that was instructive with Fraunhofer, at Munich, so that I have been able to occupy

myself with benefit there for about a fortnight." But he writes to his wife from Paris in February, 1823. "My stay here grows more and more interesting to me every day. The acquaintances I have made grow every day more cordial and intimate." He saw Biot, Fresnel, Poillet, Ampère, Arago, Fourier, Dulong and many others; such was the brilliant list of physicists there at work at Paris. He had long discussions with Ampère on his famous theory, still accepted, that magnetism consists of electric currents in the molecules—electron currents or oscillations as we should perhaps say to-day. Oersted adds, "On the 10th I was at Ampère's by appointment to see his experiments. He had invited not a few—he had three considerable galvanic apparatus ready; his instruments for showing his experiments are very complex; but what happened? Hardly any of his experiments succeeded. He is dreadfully confused and is equally unskilful as an experimenter and as a debater." This report is in strange contrast with the written records of Ampère which Maxwell has described as the work of the "Newton of Electricity," "perfect in form and unassailable in accuracy." Perhaps Ampère had had the best of an argument!

What then has been added in the last hundred years? Well, the answer to that question will depend on whether you are a so-called practical man or a theorist, whether you are most interested in the applications and practical achievements of physics or in the great principles and theories which underlie the theory and from which the practical applications necessarily arise.

The last hundred years have speeded up all human activities. It now takes days for matter to cross the Atlantic instead of weeks, as then; while messages are flashed across almost instantaneously. A hundred miles a day by coach or on horseback was a strenuous journey, a thousand miles a day by rail is to-day not formidable.

It has been argued with much force by R. A. Freeman in his "Social Decay and Regeneration" that mankind has suffered to a terrible extent by the great access of power which science has suddenly placed in its hands, and it may well be doubted if society is yet fitted to receive fresh gifts of energy from the hands of science. Moral development and social organization has lagged behind scientific progress. Human nature is stable and ill fitted to adapt itself to changes of the magnitude and variety of the last

three generations. The resultant instability of modern conditions has shown itself to the greatest extent where the attempted assimilation has been most rapid and ill digested. Petrograd stands out as a prominent and inconceivable wreck, through the mirage of a prostrate Russia.

When we turn our attention to the intellectual achievement of physics we see a far more attractive picture. The last hundred years have seen the almost complete development of the science of electricity.

The great principle of the conservation of energy established by the insight of Joule, Kelvin, Helmholtz and others, stands together with the Second Law of Thermodynamics as the main prop of all physical conceptions. The isolation of the electron, the discovery of its properties, experiments with alpha and röntgen rays and immense developments in modern spectroscopy are illuminating a vivid conception of the structure of the atom. The present century is responsible for the new branch of physics, and in this very place Rutherford delved deep and built high in radioactivity, and we are all gathered together at a "veritable shrine," already venerated as such. We are passing to a new outlook where energy becomes dominant, so that not only does matter appear to be energy, but space, linked with time from which it is inseparable, is regarded as a continuum of energy mainly.

Most important of all is our revision of fundamental conceptions on a more comprehensive scale, in accord with the general scheme of the universe of which we are denizens, embraced in the fascinating and far-reaching Principle of Relativity.

Those only who have specialized in modern physics are familiar with the strange elusive problems embraced in the Quantum Theories of Energy.

An atomistic theory of matter is easy to conceive. A corpuscle of electricity, now called an electron, with well-marked properties, electric and magnetic, is not too obscure. But bundles of energy, or quanta, of magnitudes varying with and proportional to the frequency of the propulsive electromagnetic vibrations present formidable obstacles to the human intelligence, and yet some such entities pervade modern research, and are to-day most fruitful of actual philosophical progress.

I wonder what my successor, lecturing here one hundred years hence, will be saying about relativity and about quanta!

Utilization of Nitre Cake. W. H. H. NORRIS (*Jour. Soc. Chem. Ind., Trans.*, 1921, xl, 208-212) describes a new process for the utilization of nitre cake. It is based upon certain phase relations. Nitre cake is treated, under definite conditions, with a solution containing sodium sulphate and ammonium sulphate; anhydrous sodium sulphate precipitates, and a strongly acid liquid is formed. The process is continuous. The mother liquor and washings, from the ammonium sulphate of the preceding cycle, have the following composition in pounds: Ammonium sulphate, 5800; sodium sulphate, 2295; sulphuric acid, 318; water, 5800. They are heated to a temperature of 90° C.; and 5550 pounds of granulated nitre cake are added. Anhydrous sodium sulphate weighing 3880 pounds precipitates, and 1670 pounds of sulphuric acid pass into solution. The liquid is diluted with 6700 pounds of water, and is used for the absorption of the ammonia produced by a gas plant. The resulting liquid is concentrated in a vacuum pan to remove 9300 pounds of water; 2240 pounds of ammonium sulphate deposit are collected, subjected to centrifugation, and washed. The combined mother liquor and washings are used for the treatment of another batch of nitre cake, and the entire cycle is repeated. The anhydrous sodium sulphate is of good color, contains 95 per cent. sodium sulphate, 0.04 to 0.15 per cent. sulphuric acid, and 0.24 to 0.39 per cent. ammonium sulphate; the washed salt contains less than 0.05 per cent. iron, and is of value in the manufacture of plate glass. The ammonium sulphate contains over 24 per cent. ammonia, and approximately 95 per cent. absolute ammonium sulphate.

J. S. H.

Physicochemical Study of Invertase.—The enzyme invertase, also called sucrase or saccharase, is a catalyst which produces hydrolysis of sucrose or cane sugar into invert sugar, an equimolecular mixture of *d*-glucose and *d*-fructose. The silver ion acts as a poison to this catalyst. H. VON EULER and O. SVANBERG (*Arkiv för Kemi, Mineralogi, och Geologi*, 1921, viii, No. 12, 1–13) obtained highly purified invertase by prolonged dialysis. They then determined, in the electrometric way, the dissociation constant of the silver derivative of the enzyme, and of the silver derivative of each of the following compounds, all of which are closely related to the nucleoproteins: Guanylic acid, inosinic acid, adenylic acid, guanosin, adenosin, caffeine, guanine, and uric acid. Their results indicated that a complex group, which contains phosphorus and is similar to the complex groups found in the nucleoprotein molecule, is present in invertase. This conclusion was supported by the fact that the invertase preparation did not contain sufficient bases to combine with its phosphoric oxide, hence the latter must exist in organic combinations.

J. S. H.

A STUDY OF SEWAGE AND TRADE WASTES AT BRIDGEPORT, CONNECTICUT.*

BY

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Bureau of Chemistry, U.S. Department of Agriculture.

INDUSTRIAL wastes and sewage of cities and towns along the Atlantic seaboard usually are disposed of conveniently and at least expense by discharging them without treatment into adjacent harbors or the estuaries of rivers. The practice is of long duration and has become more or less established as a right of manufacturing establishments and of cities situated along the Atlantic coast.

While the uncontrolled dumping of untreated sewage and trade wastes into bays, harbors and estuaries along the coast is an inexpensive and natural method of disposal, marked disadvantages of this custom are becoming apparent. Among these are the gradual conversion of naturally beautiful harbors and estuaries into discolored and foul-smelling pools, the interference with sports and recreation along the shores, and, what is more important in the economic life of the nation, the destructive effect on fish and shell fish life, thereby affecting seriously the natural yield as well as possible future development of an important food supply. Where questions of expense and convenience are involved, esthetic sentiment does not receive the consideration which it perhaps deserves. It is difficult to arouse general public interest in a project to keep our waterways clean when the argument is based primarily upon esthetic considerations and when the true economic possibilities can be proven only indirectly, or must be assumed.

The destructive effect of industrial wastes on fish and shell fish life is, however, a very practical problem. Oysters are said to spawn more vigorously in the protected, shallow, warmer waters of the unpolluted harbors and estuaries of the rivers than they do out on the open beds, while the polluted water from some of the harbors, we are informed by the Bureau of Fisheries, will

* Contribution from the Water and Beverage Laboratory, Bureau of Chemistry, U. S. Department of Agriculture. Communicated by Dr. Skinner.

cause the death of oyster larvæ almost at once when placed in it. The effect of the wastes on the migration and spawning of fish is also said to be of importance. These factors are gradually receiving increased attention, and the question of the exercise of some sort of control over the disposal of sewage and industrial waste is being raised more frequently by those interested in the conservation of our natural food resources.

BRIDGEPORT, CONNECTICUT, HARBOR; AND YORK RIVER, VIRGINIA.

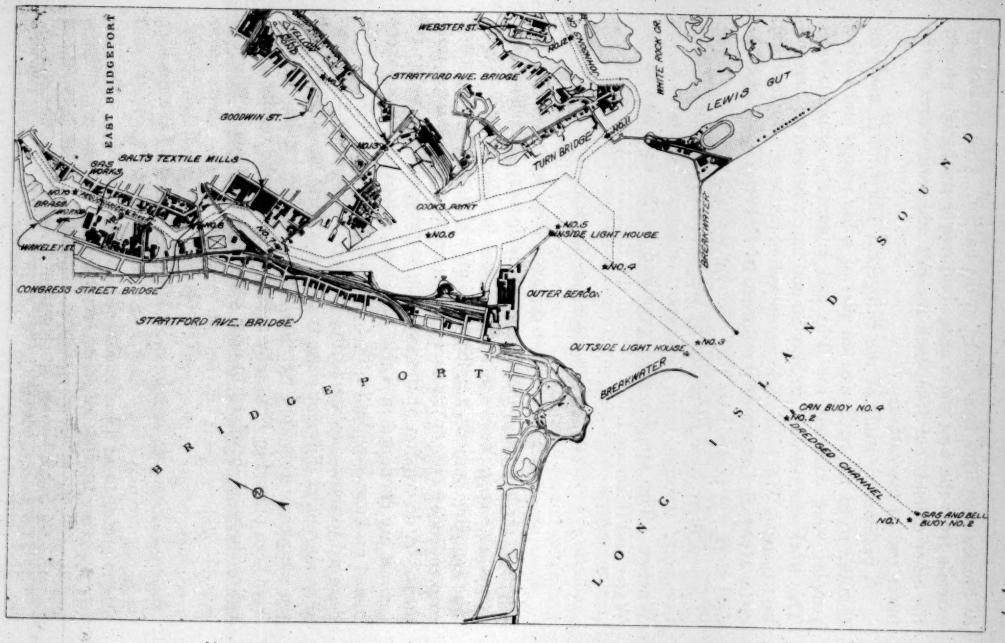
The Bureau of Fisheries requested the coöperation of the Bureau of Chemistry in a study of industrial wastes at West Point, Virginia, on the York River, and at Bridgeport, Connecticut, on Long Island Sound. This work was instituted primarily to ascertain, if possible, by chemical means, the extent of the pollution with sewage and industrial wastes, and its relation to the fish and shell fish industries. These places were selected for study because they offered peculiar advantages for investigation of trade waste, and because numerous complaints had been made by oystermen and others regarding existing conditions. A fairly complete study of the York River situation was made and a report of the work will be submitted at a later date.

BRIDGEPORT, CONNECTICUT, HARBOR.

The population of Bridgeport, Connecticut, is given in the 1920 census as 143,152.

The harbor proper (see Fig. 1) is approximately one and one-half miles long, one-half mile wide at the upper end and one mile wide at the harbor entrance. A channel dredged to from 17 to 21 feet in depth extends down the harbor and about one mile beyond the outside breakwaters. The Pequonnock River, a small stream, flows into the upper end of the harbor, and a small shallow body of water, known as the Yellow Mill Pond, is connected with the harbor at the upper end. An estuary, known as Johnson's Creek, is located at the lower end of the harbor. This estuary is not of much importance from the standpoint of a sanitary survey, the only industrial establishment of any magnitude on this stream being the Lake Submarine Corporation. The city sewage and numerous industrial effluents are discharged into the Pequonnock River, the Yellow Mill Pond and harbor proper. Two series

FIG. 1.



Map of Bridgeport, Conn., narbor and environs, showing location of sampling stations.

of breakwaters have been placed in the harbor. One breakwater extends from the west shore about midway of the harbor proper, the other extends out from both the east and west shores at the lower end of the harbor.

A preliminary study of the Bridgeport Harbor was made on October 16 to 18, 1918, and samples of trade wastes and harbor samples were collected for field and laboratory examinations. This work was repeated on May 20 and 24, 1919, and on September 16, 1919, Mr. J. W. Sale, accompanied by Dr. E. P. Churchill or by Mr. J. S. Gutsell, of the Bureau of Fisheries, conducted the field work. A power boat for collecting samples and the services of an engineer were furnished by the Connecticut Oyster Farms Company, through their Bridgeport manager, Captain C. S. Wheeler. The samples were examined in the field and in the Water and Beverage Laboratory of the Bureau of Chemistry.

SEWAGE IN BRIDGEPORT HARBOR.

Four series of samples were collected from both near the top and near the bottom in the channel of Bridgeport Harbor proper, Johnson's Creek, Yellow Mill Pond, and the Pequonnock River. The sampling stations, the locations of which are shown on the map appended (Fig. 1) were from one-eighth to one-quarter mile apart. A description of the sampling stations and other data of interest are contained in Table 1. Dissolved oxygen was determined on these samples in the field. This part of the work was designed to show the extent of the sewage pollution of the harbor and environs at different seasons of the year. One hundred and ten samples of this kind were collected, and the results obtained have been plotted in Fig. 2, by means of which the results of the entire study of dissolved oxygen may be seen at a glance.

TABLE I.

Location of Sampling Stations, Bridgeport (Conn.) Harbor and Environs.

Station No.	
1	Long Island Sound off gas and bell buoy No. 2.
2	Long Island Sound off can buoy No. 4.
3	Off outside lighthouse, between breakwaters.
4	Off beacon between outside and inside lighthouses.
5	Off inside lighthouse.
6	Off Cook's Point.

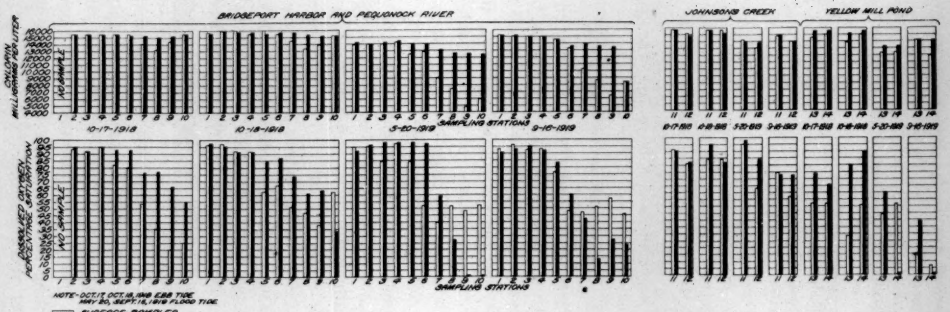
7 At Stratford Ave. Bridge, Pequonnock River.
8 At Congress St. Bridge, Pequonnock River.
9 Off gas works, Pequonnock River.
10 Off Wakeley St., Pequonnock River.
Johnson's Creek.
11 At turn bridge.
12 Off Webster St.
Yellow Mill Pond.
13 At Stratford Ave. Bridge.
14 Off Goodwin St.

On October 17, 18 (1918) the tide was receding and on May 20 and Sept. 16 (1919) the tide was flooding when the samples were collected. On Oct. 17, 18 (1918) and May 20 (1919) the temperature of the samples varied from 12°C. to 17°C. and on Sept. 16 (1919) from 19.5°C to 22°C. The deep samples were collected at depths of from 8 to 30 feet. The percentage of sodium chloride in all samples varied from 0.78 to 2.70.

DISCUSSION OF OXYGEN DATA.

For the purpose of considering the dissolved oxygen content, the harbor environs may be divided arbitrarily into five sections; namely, the wind swept portion in Long Island Sound extending about a mile beyond the outside breakwaters, the lower harbor, including the area between the outside and inside breakwaters, the upper harbor including the area between the inside breakwater and Stratford Avenue bridges, the estuary of the Pequonnock River, the Yellow Mill Pond, and the estuary of Johnson's Creek. Fourteen samples were collected from the area beyond the outside breakwater at sampling stations numbers 1 and 2. This section is beyond the area materially affected by sewage and fairly represents the character of Long Island Sound water in this vicinity. The average dissolved oxygen content of both surface and deep samples was 95 per cent. saturation, maximum 97 per cent., and minimum 90 per cent. These figures may be accepted as a standard with which to compare the dissolved oxygen content of the other areas. Proceeding into the harbor, the area between the outside and inside breakwaters, which has been designated above as the lower harbor, contained an average of 93 per cent. saturation dissolved oxygen, maximum 98 per cent., minimum 84 per cent, 16 samples from the stations numbered 3 and 4 being represented. Proceeding further up the harbor, the area between the inside breakwater and Stratford Avenue Bridge, which has been called the upper harbor, contained an average of 77 per cent. saturation dissolved oxygen, maximum 98 per cent., minimum 48 per

FIG. 2.



Graphic presentation of dissolved oxygen and of chlorin content of samples.

cent., 16 samples from stations numbered 5 and 6 being represented. The estuary of the Pequonnock River contained an average of 13 per cent. saturation dissolved oxygen, maximum 77 per cent., minimum 00. per cent., 32 samples from stations numbered 7, 8, 9 and 10 being represented. This area called the estuary of the Pequonnock River is in reality a part of the upper harbor. The water is salty, containing on an average of 2.0 per cent. sodium chloride, the area beyond the outside breakwater containing about 2.5 per cent. sodium chloride. The Yellow Mill Pond contained an average of 44 per cent. saturation dissolved oxygen, maximum 90 per cent., minimum 00. per cent., 16 samples from stations numbered 11 and 12 being represented. Johnson's Creek contained an average of 81 per cent. saturation dissolved oxygen, maximum 97 per cent., minimum 56 per cent., 16 samples from stations numbered 13 and 14 being represented. As the sources of pollution are approached the dissolved oxygen content decreases as follows, 95, 93, 81, 77, 44 and 13 per cent., these figures representing averages of numerous samples, collected at different seasons and at varying tides. Dissolved oxygen was absent part of the time in the estuary of the Pequonnock River and in the Yellow Mill Pond. When dissolved oxygen is exhausted in water containing much organic matter, active decomposition sets in with the production of hydrogen, methane, sulphide of hydrogen, etc., which condition is characteristic of sewage septic tanks. At various points in the Pequonnock River and the Yellow Mill Pond, bubbles of gas breaking on the surface of the water were noticed, indicating septic decomposition of the sewage sledge deposited on the bottom. The estuary of the Pequonnock River is nearly always highly discolored, especially in the vicinity of Salts' Textile Mills, from which flow great quantities of purple effluents. Above Stratford Avenue Bridge in the Pequonnock River and above Stratford Avenue Bridge in Yellow Mill Pond, visible evidence of fæcal pollution is abundant. During the summer months the odor from these waters is nauseating. However, as mentioned above, a very marked improvement in the appearance and character of the water is evident as one proceeds out into the lower harbor. This improvement undoubtedly is due to the fact that the organic matter dissolved and suspended in the city sewage is rapidly precipitated out when the sewage mixes with the salt water of the Pequonnock River estuary, the upper harbor and the Yellow Mill

Pond. The precipitated material which is highly decomposable, remains on the bottom in the vicinity of the sewer outlets in a relatively small area instead of being distributed over a larger area where it might come into contact with and be purified by the highly oxygenated sea water coming in with the tides from Long Island Sound. This precipitation of dissolved and suspended organic matter in sewage by salt water is an interesting physical phenomenon which we have discussed quite fully in a previous publication,[1] and which must be given consideration in connection with a study of Bridgeport Harbor. The sewage deposit in the Pequonnock River estuary and in the Yellow Mill Pond is quite evidently not swept out to sea, as one might be led to believe by an inspection of the chart of the harbor; on the contrary, such sewage sludge as may be dislodged from the bottom is carried back and forth within a limited area by the action of the tides. It seems probable that the inside breakwater at sampling station 5 (see Fig. 1) tends to prevent the outflow of the polluted water of the inner harbors, but it is believed that the difference in quality between the waters of the Pequonnock River estuary and the Yellow Mill Pond and those of the lower harbor and beyond is due chiefly to the precipitation by salt water of sewage solids in a coherent mass near the sewer outlets.

There is, therefore, a marked contrast of conditions, from a sewage standpoint, rather than a gradual change of condition. This is shown by the drop in dissolved oxygen content from 77 per cent. to 44 per cent., and 13 per cent., respectively, and by the physical appearance of the areas compared. The estuary of the Pequonnock River and the Yellow Mill Pond at the time of this investigation were veritable septic tanks, whereas the harbor outside of Cook's Point (station No. 6) was in fairly satisfactory condition. It should be stated that since this investigation was made, a sewage disposal plant, which clarifies the sewage by means of fine screens, has been installed. This plant is designed to take care of the sewage from the section of the city lying west of the Pequonnock River, which has an estimated population of 100,000, and will ultimately reduce the number of sewage outlets from about 57 to 2. The operation of this plant should materially improve the objectionable conditions described above.

[1] The Vertical Distribution of Dissolved Oxygen and the Precipitation by Salt Water in Certain Tidal Areas, J. FRANKLIN INST., Dec., 1917.

INDUSTRIAL WASTES IN BRIDGEPORT HARBOR.

The analyses of industrial wastes flowing into Bridgeport Harbor from nine sewers are set forth in Table 2. Altogether, the effluents from twenty-one sewers were analyzed, but the effluents from only nine sewers were found to contain unusual ingredients. A description of the location of the twelve sewers not included in Table 2 is as follows: (10) At Berkshire Coal Docks, on west side of Pequonnock River, about 100 yards below dam; (11) off plant of Armstrong Manufacturing Company, on east side of river, between Maple Street and Barnum Avenue; (12) Housatonic Avenue, sewer on west side of Pequonnock River at Washington Avenue Bridge; (13) weir of Bridgeport Illuminating Company, on west side of river; (14) Congress Street sewer under Congress Street Bridge, on west side of Pequonnock River; (15) ten-inch sewer underneath piling off Salts' Textile Mills, on east side of Pequonnock River; (16) Remington Arms Company's weir at upper end of Yellow Mill Pond; (17) off plant of Holmes and Edwards Silver Company, at upper end of Yellow Mill Pond; (18-19) two six-inch sewers off British-American Munition Works, on northwest side of Yellow Mill Pond; (20) large sewer off British-American Munition Works, on northwest side of Yellow Mill Pond; (21) weir at American Tube and Stamping Company (steel works), in Bridgeport Harbor, below Stratford Avenue Bridge.

The effluents from the nine sewers, the composition of which is set forth in Table 2, are objectionable because of the presence of metals, color, or acid. Those containing relatively large amounts of these ingredients are seven in number, and appeared to come from two sources only; namely, the manufacturing establishments of the Bridgeport Brass Works, situated on the west side of the Pequonnock River estuary, and the Salts' Textile Mills, situated on the east side of the Pequonnock River estuary. Great variation is to be expected in the composition of individual samples collected directly from sewer outlets, and the data obtained on the seven sewage effluents mentioned above do vary greatly. At times, relatively large quantities of copper, zinc, acid, color, etc., are shown to be present, while at other times these objectionable ingredients are either absent or present in greatly reduced amounts.

TABLE II

Analysis of Sewer Samples, Pequonnock River and Yellow Mill Pond, Bridgeport, Conn.

No.	Location	Total Solids Ignited	Alkalinity as Bicarbonate HCO_3	Acidity as Sulphuric Acid H_2SO_4	Milligrams per liter except chloride: Iron Fe	Copper Cu	Aluminum Al	Manganese Mn	Lead Pb	Zinc Zn	Arsenic As
1-3	Pequonnock River: Off Bridgeport Brass Works (west side of river). About one hundred yards below new bridge. Three sewers close together.										
	2 ft. dia. Oct. 16, 1918, W.L. 994	21915	114		2.5(*)	0.0		0.0	1.3	26.	0.02
	May 24, 1919, W.L. 755	56	20		.5	0.0	1.4	0.0	0.0	0.	0.00
	Sept. 16, 1919, W.L. 924	15288		304	3.0	240.2	3.3	.08	0.0	40.	
	8 in. dia. Oct. 16, 1918, W. L. 995	288		46	.4(*)	65.		0.0	1.9	9.	0.00
	Sept. 16, 1919, W.L. 925	19620	104		.6	.1	.9	.06	0.0	0.	
	6 in. dia. Oct. 16, 1918, W.L. 996	800		232	11. (*)	18.5		.08	1.0	0.	0.00
4-5	Pequonnock River: Off Bridgeport Brass Works (west side of river). Two fourteen-inch sewers located one hundred yards below the sewers mentioned above.										
	North Oct. 16, 1918, W.L. 997	578	28.		1.6(*)	0.0		.2	.31	21.	0.00
	Sept. 16, 1919, W.L. 928	456	30.		19.5	118.0	9.9	.08	0.4	83.	
	South Oct. 16, 1918, W.L. 998	1025	68.		62.4(*)	18.3		0.0	.27	101.	0.00
	Sept. 16, 1919, W.L. 927	396	24		2.5	2.4	1.7	.17	0.0	0.	
6	Pequonnock River: Twelve-inch sewer a few feet below Congress St. Bridge (east side of river).										
	Oct. 16, 1918, W.L. 1003	471			1. (*)	2.0		0.0	0.0	0.	0.03
	May 24, 1919, W.L. 756	147	139		2.5	0.0	4.8	.02	0.0	0.	0.00
	Sept. 16, 1919, W.L. 926	892	232		3.8	.2	3.8	0.0	0.0	0.	

7	Pequonnock River. Surface sewer located about twenty-five yards below preceding sewer, held in place by posts.										
	Oct. 16, 1918, W.L. 1004....	1188	273(‡)		14.1	0.0		.2	0.0	0.	0.14mg.
	May 24, 1919, W.L. 757....	190	96		.6	0.0	4.8	0.0	.2	trace	.00
	Sept. 16, 1919, W.L. 929....	804	85		5.2	2.3	3.2	.08	0.0	0.	
8	Pequonnnock River: Two foot sewer off Salts' Textile Mills (east side).										
	Oct. 16, 1918, W.L. 1005....	273(†)			1.2(*)	0.0		0.0	0.0	0.	0.00
	May 24, 1919, W.L. 758....	32(†)	15		.3	0.0	.9	0.0	0.0	0.	0.00
	Sept. 16, 1919, W.L. 931....	3260(†)	85		.6	1.2	1.1	0.0	0.0	0.	
9	Pequonnock River: South end of Salts' Textile Mills (east side of river). About one hundred yards below preceding sewer.										
	Oct. 16, 1918, W.L. 1006....	533(†)			6. (*)	4.0		.2	0.0	0.	
	May 24, 1919, W.L. 759....	1321	215		3.3	0.0	3.0	0.0	0.0	0.	0.00

(*) Fe and Al as Fe. (†) Effluent colored deep purple. (‡) 40 m. p. l. CO_3 found.

EFFLUENTS IN VICINITY OF THE BRIDGEPORT BRASS WORKS, AND ELECTROLYTIC RECOVERY OF COPPER AND SULPHURIC ACID.

The effluents from the sewers in the vicinity of the Bridgeport Brass Works were found to contain acid, copper, and zinc in relatively large amounts. One sample contained 240 milligrams of copper per liter, another 118 milligrams per liter, another 65 milligrams per liter, and two others 18 milligrams per liter, each. One sample contained 101 milligrams per liter of zinc, and five others 83, 40, 26, 21 and 9 milligrams per liter, respectively. Three samples contained mineral acid, which, expressed as sulphuric acid, amounted to 304, 232 and 46 milligrams per liter, respectively.

The other samples in this group were alkaline. Several samples contained small quantities of lead and manganese. Where the samples were acid, the copper and zinc were present chiefly in solution in the effluent. Where the samples were alkaline, the copper, zinc and lead were found chiefly in the sediment which settled out in the bottles after the samples were collected. For example, in W. L. No. 924 (see Table 2, Nos. 1-3) the filtered sample, which was acid, contained 240 milligrams per liter of copper and 40 milligrams per liter of zinc. The sediment contained only 0.2 milligrams per liter copper and no zinc. However, in W. L. 928 (see Table 2, Nos. 4-5) the filtered sample, which was alkaline, contained no copper or zinc, and the sediment derived from one liter of effluent contained 118 milligrams copper, 83 milligrams zinc and 0.4 milligrams lead. In Table 2 the analyses of the supernatant water and of the sediment have been reported together. A composite sample, consisting of six individual top and bottom samples, which were collected from the channel of the Pequonnock River (1, off Gas Works; 2, at Congress Street Bridge; and 3, at Stratford Avenue Bridge), was analyzed. 0.8 milligrams per liter copper found. It is an interesting fact that the oysters grown in the vicinity of Bridgeport Harbor have a puckery taste and a greenish color. With regard to the mineral acid present in these effluents, it should be noted that all of the 110 samples of water collected from the Pequonnock River estuary, and the harbor, were alkaline, showing that the quantity of acid in the effluents was not sufficiently large to overcome the natural alkalinity of the harbor water during the period of observation.

The metals were removed from the acid samples quite effectively (over 99 per cent.) by adding soda ash or lime water in slight excess in the acid followed by filtration. This appears to be a fairly simple method of chemical treatment. In the preliminary investigation, we did not inspect the factories or study the method of production of the objectionable effluents, but in the summer of 1920, the question of the recovery by electrolysis of copper and acid from the effluents was taken up with the officials of the Bridgeport Brass Company, and in January, 1921, the installation of an experimental recovery tank was completed. During the spring of 1921 a number of experimental runs were made. We are indebted to the Bridgeport Brass Company for the following data on this reclaiming tank.

The acid liquor from four pickling tanks is subjected to continuous electrolysis in a lead-lined recovery tank, divided into three compartments. The liquor is circulated through the five tanks by a motor driven pump. The electrodes consist of 96 sheets of copper and lead, each of which are 28 inches long and 30 inches wide, set longitudinally in the three compartments of the tank. In an experimental run May 5, 6 and 7, 1921, two compartments of the tank were connected in series, the copper plates used having an area available for deposition of copper of 300 square feet. The liquor was circulated by the pump at the rate of 4 gallons per minute, the power delivered to the motor generator was 15,400 watts (230 volts x 67 amperes). The power delivered to the recovery tank was 6600 watts (5½ volts x 1200 amperes), showing an efficiency of 42.8 per cent. One hundred and sixteen pounds of copper and 180 pounds of acid were recovered during the run of 44 hours. The cost of the current delivered to motor generator at 2 cents per kilowatt hour was $13.55. The motor driven pump used $1.32 worth of current, making total cost of current used during the test $14.87. The 116 pounds of copper recovered at 12 cents a pound, was worth $13.92, and the 180 pounds of regenerated sulphuric acid, at 1¾ cents per pound, was worth $3.15, a total of $17.07. The profit, therefore, on this experimental run, without figuring in overhead expense, amounted to $17.07 less $14.87, or $2.20. The proposition viewed from a strictly commercial standpoint is therefore not very encouraging, although it is hoped that a better utilization of the current, which is the chief item of expense, can be obtained. No great trouble is antici-

pated in overcoming certain other difficulties which have been encountered; namely, fuming of the solution and the overflow when bundles of tubes are dipped in the pickling tanks. Reclaiming processes of a similar nature are in operation in the plants of the Chase Companies, Inc., of Waterbury, Connecticut, and of the Westinghouse Electric and Manufacturing Company, of East Pittsburgh, Pennsylvania. At the latter place, one pound of copper per kilowatt hour is recovered.[2]

The effluent containing copper and acid which flows into the harbor is derived chiefly from wash tanks in which bundles of tubes which have been pickled are rinsed off. The effluent contains some overflow from the pickle tanks also. The effluent from four wash tanks amounts to about 5400 gallons per hour which contains, according to the officials of the Bridgeport Brass Works, 9 pounds of copper and 40 pounds of sulphuric acid.

EFFLUENTS IN VICINITY OF SALTS' TEXTILE MILLS.

Four of the effluents from the vicinity of Salts' Textile Mills coming from two sewers were dyed a deep purple. These effluents stained the harbor for a long distance, and are of greater importance from the standpoint of appearance of the harbor than any other effluent. There is probably no other type of effluent that arouses the resentment of people so quickly as that which is highly colored, even though it may be harmless. The actual quantity of dye present may be very small, yet the effect very marked, as is the situation at Bridgeport. An effluent of this type ought to be decolorized before being discharged into the harbor. A small quantity of bone charcoal was placed in a filter, some of the effluent passed through in the cold and a water white filtrate was obtained. Treatment with nascent chlorine or calcium hypochlorite was not so effective in decolorizing the effluent.

CORRECTIVE MEASURES.

Very small quantities of unusual ingredients are present in some of the other samples. However, those discussed above are so much worse than the others which were sampled that they have been given first consideration. If the trade wastes which are discharged into the harbor by the Bridgeport Brass Company and by

[2] *Transactions Am. Electrochem. Soc.*, 32 (1917) (Preprint).

the Salts' Textile Mills were treated before being dicharged in the harbor, undoubtedly the conditions other than those due to sewage would be greatly improved. For this reason we hope that the experiments in progress on the recovery of copper and acid may prove to be sufficiently profitable to warrant purification of all effluents which contain copper or acid.

CONCLUSION.

The need of an adequate sewage treatment plant at Bridgeport, Connecticut, is clearly shown by the data contained in this report, and it is gratifying to those interested in preserving the natural beauty of the harbors along the Atlantic Seaboard and in the conservation of our natural resources, that the city of Bridgeport is taking active steps in remedying the conditions set forth herein. The coöperation of one of the principal industrial plants which produces noxious effluents has been secured also, with a view to installation of adequate effluent treatment.

Chrome Ores of Southeastern Pennsylvania and Maryland.—The deposits of chromite in the serpentine belt of southeastern Pennsylvania and eastern Maryland have been studied by ELEANORA B. KNOPF (*U. S. Geological Survey Bulletin 725 B*, 85–99 (1921)). Chrome ore was discovered in Maryland as early as 1827; and that state and Pennsylvania furnished the world's supply of the ore until 1860. Since that year Turkey has been the chief source of chromite. The chrome mining industry of the eastern states has been dormant since 1882; and the domestic production of this ore has practically been confined to the Pacific coast. In the deposits of Pennsylvania and Maryland, chromite occurs as rock ore and in alluvial sand. The rock ore may be in either massive or granular form, and is found in serpentinized pyroxenites and peridotites. The ore is of good quality, containing from 48 to 63 per cent. of chromium sesquioxide. The ore occurs at intervals in a belt which extends for a distance of 50 miles from the southwestern corner of Chester County, Pennsylvania, to the neighborhood of Baltimore, Maryland. At least twelve abandoned mines and prospects exist along this belt; they were closed because of a decline in price, rather than because of exhaustion of the ore. During the war, on account of the demand for chromite, two of these mines were reopened, and preparations were made to concentrate the disseminated ore. Isolated deposits of chromite also occur in Delaware County, Pennsylvania, and have been worked to a small extent. J. S. H.

Some Mechanical Curiosities Connected with the Earth's Field of Force. W. D. LAMBERT. (*Am. J. Science*, Sept., 1921.)—If the earth were a sphere of uniform density with no rotation about any axis, its field of force, due to attraction according to the inverse square law, would be a rather simple affair. It is not a sphere but a mildly bulging spheroid and it has, to complicate matters further, a rotation about an axis, which brings into play at every point a force that must be compounded with that other force there caused by attraction before we can know the intensity and the direction of the force of gravity at that point. The force of gravity is the resultant of the two forces mentioned.

The author discusses the form of equipotential or level surfaces outside of the earth. At a considerable distance above us the level surface is shaped somewhat like that made by placing two equal and thick plano-convex lenses with their flat surfaces in contact. There is a distinct edge to this surface above the equator of the earth. Lower down the level surfaces cease to display this edge. They are in addition not similar to one another, being farther apart at the equator than at the poles. A level surface 1000 metres above sea-level at the equator is about 5 metres nearer to the level of the sea at the pole. A concrete illustration of this is the fact that the northern end of the surface of Lake Michigan, taken as undisturbed by waves, temperature or flow, is 8 cm. nearer to sea-level than the southern end.

A vertical line at any point is the direction of the force of gravity there and is perpendicular to the level surface passing through the point. Such a line, cutting all the level surfaces it meets orthogonally, is not a straight line, but is curved with its convexity toward the equator. "The astronomical latitude of a place is defined as the angle which the vertical at that place makes with the plane of the equator. Evidently then the latitude at the top of a high tower is greater than the latitude of its base."

If the continents are regarded as huge floating bodies projecting above the surfare of the supporting, denser liquid, then there will be a slow motion toward the equator, generally accompanied by a twist of the direction of greatest length. The torsion balance of Eötvös not only indicates but even measures such peculiarities in the gravity field of force as tend to shift the position of continents. Certain results obtained by the use of this instrument are closely dependent on the density and form of the adjacent portions of the earth, and accordingly the balance has been used to search for lignite in Austria.

"The form of a level surface near the surface of the earth has been appropriately compared to that of a withered apple, which, considered as a whole, has a regular curvature, but which, taken in detail, is characterized by minute elevations and depressions."

G. F. S.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

SPECIFIC VOLUME OF LIQUID AMMONIA.[1]

By C. S. Cragoe and D. R. Harper, 3rd.

[ABSTRACT.]

THE paper is a detailed description of a laboratory measurement, with very high precision over a considerable temperature range, of the specific volume of pure anhydrous liquid ammonia under the pressure of saturation at each temperature. This is one of the series of measurements of the physical properties of pure ammonia in progress at the Bureau of Standards for the purpose of compiling a standard ammonia table for use by the refrigeration industry.

Four glass picnometers of about 5 cm^3 capacity were calibrated with water and mercury to an accuracy of a few parts in a hundred thousand. The temperature coefficient of expansion of the glass was obtained from a determination with an interferometer of the coefficient of linear expansion. The expansion of the picnometers with pressure was determined experimentally, using a liquid of known compressibility.

Seven samples of ammonia, purified by four somewhat different methods, were used in these picnometers, and volume measurements were made throughout the temperature interval − 78 to + 100° C. Special tests showed less than 1 part in 10,000 (by volume) of noncondensing gases present, and less than 0.01 per cent. (by weight) of other impurities.

The picnometers, after being partially filled with liquid ammonia, were immersed in a thermoregulated bath, the temperature of which could be maintained constant to about 0.01° C. Readings at various temperatures were then taken of the position of the ammonia meniscus in the calibrated capillary of the picnometer. Readings were taken at 5-minute intervals for a period of about ½ hour in order to insure equilibrium.

The total mass of ammonia in each picnometer during any series of measurements was always determined afterwards by

* Communicated by the Director.

[1] Scientific Paper, No. 420.

weighing the picnometer filled with ammonia, then breaking the tip and reweighing, filled with dry air. In the latter series, the total mass was also determined before filling the picnometers, using a small auxiliary bulb soldered to a steel valve. The mass so determined agreed with the subsequent determination to better than 1 part in 10,000.

The picnometers were so designed as to make the mass of the vapor phase small in comparison with the total mass of ammonia. The values of the specific volume of the saturated vapor which enter as a correction term were obtained from preliminary measurements made at this Bureau. In the interval + 50 to + 100° C., these values were obtained by extrapolating the mean diameter of the temperature-density dome.

An empirical equation was found which represents closely the results of the temperature range covered experimentally and also conforms to what is known about the behavior of liquids at the critical temperature. The average deviation of all the measurements from this equation is 1 part in 10,000, and the maximum deviation is 4 parts in 10,000.

As a final result, the specific volume of saturated liquid ammonia is expressed by the equation

$$U = \frac{4.2830 + 0.813055\sqrt{133-\theta} - 0.0082861\,(133-\theta)}{1 + 0.424805\sqrt{133-\theta} + 0.015938\,(133-\theta)}$$

where U is expressed in cubic centimeters per gram and θ in degrees Centigrade.

WAVE-LENGTHS LONGER THAN 5500 A IN THE ARC SPECTRA OF YTTRIUM, LANTHANUM, AND CERIUM AND THE PREPARATION OF PURE RARE EARTH COMPOUNDS.[2]

By C. C. Kiess, B. S. Hopkins and K. C. Kremers.

[ABSTRACT.]

PART I of this paper, by C. C. Kiess, contains the results derived from a study of the yellow, red, and infra-red regions of the arc spectra of yttrium, lanthanum, and cerium. Spectrograms were photographed, with the dyes pinacyanol, kryptocyanin, and dicyanin. The chemicals studied were derived from two sources:

[2] Scientific Paper, No. 421.

From the University of Illinois were obtained the oxalates of yttrium and lanthanum and the oxide of cerium, prepared under the direction of Professor B. S. Hopkins; from Eimer and Amend were obtained the chlorides of yttrium and lanthanum and the nitrate of cerium, the last being a Kahlbaum preparation. In addition to spectrograms of the above named materials secured with the concave grating spectrograph of the Bureau of Standards there were available for measurement two spectrograms made by Doctor Eder, of Vienna, with his grating spectrograph. These spectrograms were of yttrium and cerium salts prepared by C. Auer von Welsbach. The tables submitted herewith contain about 175 lines in the spectrum of yttrium, 410 in the spectrum of lanthanum, and about 1700 in that of cerium. Many of the wavelengths are those of heads of bands which are prominent in the spectra of yttrium and lanthanum.

Part II of this paper, prepared by Professors B. S. Hopkins and H. C. Kremers at the University of Illinois, describes the methods used in purifying the samples of yttrium, lanthanum and cerium, which were used in this work.

The rare earth compounds prepared at the University of Illinois were derived from two general sources: (1) 182 kilograms of sodium rare earth sulfate, donated by the Welsbach Company, from which were extracted compounds of cerium, samarium, lanthanum, neodymium and gadolinium; (2) 10 kilograms of Norwegian and 100 kilograms of Texan gadolinite, together with 55 kilograms of purchased oxalate which had been prepared from xenotime, from which were extracted compounds of yttrium, dysprosium and erbium.

From the Welsbach residue cerium was removed by boiling the solution to precipitate basic ceric nitrate. This was purified by repeating this process and by several precipitations with oxalic acid.

After the removal of the cerium, samarium was concentrated at the soluble end of a magnesium double nitrate series of fractional crystallizations. The magnesium double nitrate fractionation was repeated first in water and then in 30 per cent. nitric acid. Bismuth was now added and the fractionation continued until the samarium was free from other rare earth salts.

Lanthanum, which concentrated at the insoluble end of the magnesium double nitrate series, was freed from praseodymium and neodymium by crystallization of the double ammonium

nitrate, this process being continued until all absorption bands disappeared from the solution.

For the preparation of neodymium the middle fractions of the magnesium double nitrate series were selected and the fractional crystallization continued until lanthanum and praseodymium were concentrated at the insoluble end while samarium, gadolinium and europium were found at the soluble end. The middle fractions contained very pure neodymium.

Gadolinium was obtained from the fractions at the soluble end of the magnesium double nitrate series. Samarium was nearly all removed by adding bismuth magnesium nitrate. Further purification was accomplished by several precipitations with oxalic acid, but the best gadolinium so far prepared contains a small amount of samarium and terbium and traces of europium.

The yttrium group compounds were prepared mainly from gadolinite which was pulverized, dissolved in HCl, the silica removed and rare earths precipitated by oxalic acid. The oxalates were converted to sulfates and the cerium group elements removed by the addition of Na_2SO_4. The yttrium group material was converted to the bromates and subjected to a long series of fractional crystallizations. Those fractions richest in yttrium were brought together and treated by the method of fractional precipitation, using K_2CrO_4 as a precipitant. The best fractions obtained by this method were then precipitated fractionally with 0.01 N ammonia. The purest materials obtained in this manner were further purified by the sodium nitrate method of fractional precipitation. The material prepared in this manner showed very faint holmium lines when viewed through a 5 cm. layer of concentrated solution; it probably contained not more than 0.005 per cent. of holmium and was reasonably free from other earths.

Dysprosium was removed from the main bromate series, fractionated further as bromates, then converted to the ethyl sulfate and fractionated for a long time in absolute alcohol. This removed the neodymium and praseodymium but there still remained some terbium and holmium, even in the best fractions.

The erbium-rich fractions were likewise removed from the main bromate series, and were further fractionated as the bromates. The fractions containing nothing but yttrium and erbium were further purified by the basic chloride method of Drossbach, the cobalticyanide method of James and Willard, and the sodium

nitrite method of Holden and James. Each of these methods concentrated the erbium but not as rapidly as the nitrate fusion method. The best erbium fractions available contained considerable yttrium and probably small amounts of other rare earths.

FRICTION AND CARRYING CAPACITY OF BALL AND ROLLER BEARINGS.[3]

By H. L. Whittemore and S. N. Petrenko.

[ABSTRACT.]

THE experiments were undertaken by the Bureau of Standards at the suggestion of the Navy Department to determine the maximum safe load and static friction under load of ball and flexible roller bearings.

Tests were made on balls of 1.00, 1.25 and 1.50 in. diameter in grooved straight races with the ratio of groove and ball radii 1.03 and 1:10; 1.04 and 1.12; 1.04 and 1.12 respectively.

The rollers 1.25 in. in diameter and 5.25 in. long were tested in flat and cylindrical races; the inner cylindrical races having radii 3.5 and 10 in. and those of the outer races 4.75 and 11.25 in. respectively.

The scleroscope hardness of balls and ball races was about 65, that of rollers 70 and of the roller races 97.

METHOD OF TEST.

A.—Friction Test.—Two balls were used in the static friction test to secure stability in the loaded condition. The test was made in a 50,000 pound Riehlé machine; the starting effort was determined with a spring balance. One of the two races was movable and rested on rollers, whose friction was determined separately. No lubricant was used during the test.

Similar tests were made on the rollers, using steel plates. In addition, the cylindrical races were fitted to a 230,000 pound Emery testing machine. Two rollers diametrically opposite were used, being held in position by retainers having the same axis of rotation as the inner race.

The torque necessary to rotate the system was determined by a spring balance acting on a lever attached to the inner race.

B.—Compression Test.—To measure the total deformation of a ball (or roller) and two races, an apparatus was designed con-

[3] Technologic Paper, No. 201.

sisting of a simple lever system. The relative displacement of the longer arms which was proportional to the total deformation of the races and the ball was measured by micrometer microscopes. A deformation of 0.00001 in. could be measured. A similar device was used in determining the deformation of rollers.

To determine the contact area between the race and the ball (or roller) an extremely thin film of lubricating oil was applied to the surface of the race, while the ball was dry. The contact surface was distinctly visible as it appeared darker than the surrounding surface. Its diameters were measured by a micrometer microscope.

CONCLUSIONS.

Friction Test.—1. The starting friction is nearly the same for both sizes of groove. The groove having the larger radius gave the lower value.

2. The ratio of starting friction to load (0.00055) is nearly the same for all three sizes of balls and increases slowly with the load (more rapidly for small size ball) until the "critical" load is reached, when the increase becomes rapid.

The critical load varies with the diameter of ball; 1300 lb. for 1.00 in.; 1700 lb. for 1.25 in. and 2200 lb. for 1.50 in. balls.

3. A similar "critical" load, 25,000 lb., was found for the larger roller bearings with a ratio of friction to load equal to 0.00075.

4. This "critical" load at which the friction began to increase more rapidly was in all cases lower than the safe load as determined by permanent deformation and as calculated from Stribeck's law.

Compression Test.—5. The results of compression tests of balls and rollers agree roughly with Hertz's theory. The differences are ascribable to inhomogeneity of the material. The total deformation of ball (or roller) and races follows pretty closely the law of a straight line.

The smaller diameter of contact area agrees with that given by Hertz's theory, the other diameter differs considerably. This is due to the fact that the major diameter of the area of contact is not as assumed by the theory, very small in comparison with the diameter of the ball.

6. The permanent set of a ball is quite negligible even for very high loads (10,000–30,000 lb.) which produce large permanent sets in the races.

7. The permanent set of the ball races increases very gradually; there is no indication of a load below which no set occurs. With a sufficiently accurate measuring device, the permanent set of races may be discovered at a very low load, below that used in actual practice. The load limit depending upon permanent set of races may be arbitrarily assumed.

8. A flexible roller does not produce any appreciable permanent set of races unless stressed beyond its proportional limit, which is about 130,000 lb. This limit tends to decrease as the radius of races increases. The breaking load of the rollers was about 190,000 lb.

RECOMMENDED SPECIFICATIONS FOR PNEUMATIC TIRES, SOLID TIRES, AND INNER TUBES.[1]

These specifications cover the requirements for pneumatic tires, solid tires and inner tubes.

They are a revision of those prepared by the Bureau and now used by the War Department, the Navy Department, the General Supply Committee, the Post Office Department, the Panama Canal, and the Treasury Department. They were recommended by the U. S. Inter-departmental Committee on Specification Standardization, June 8, 1921.

A tentative draft of the specifications was submitted to a large number of representatives of the tire industry, including the Rubber Association of America, and in the revision careful consideration was given to their recommendations.

These specifications are divided into three parts, pneumatic tires, solid tires and inner tubes, and contain a detailed description of the physical and chemical requirements.

Some of the requirements for pneumatic tires are:

The various kinds of fabrics used in the manufacture of a tire are specified as to the length of staple, weight, and tensile strength. For fabric tires the carcass must be made from standard square-woven tire fabric having 23±1 threads per inch in warp and filling. The staple must be not less than 1¼ inches long. The tensile strength of both warp and filling shall be not less than 185 lb. per inch of width. For cord tire fabric the staple shall be not less than 1¼ inches long for all sizes of tires through 5 inch, and 1½

[1] Circular, No. 115.

inches long for the larger sizes. The tensile strength is governed by a strength factor, which varies according to the size of the tire and is determined from the average tensile strength of single cords, the number of cords per inch, and the number of plies. The number of plies required depends on the size of the tire.

The friction compound shall be made from a compound containing not less than 85 per cent. by volume of the best quality, new wild or plantation rubber. The adhesion requirements are: The friction shall be not less than 15 lb. between the plies of a fabric tire and 18 lb. between those of a cord tire; 35 lb. between cushion and breaker, and between tread and breaker; 20 lb. between cushion and carcass; and 14 lb. between sidewall and carcass for fabric tires and 18 lb. for cord tires.

The width and weight of fabric used for the breaker strip depend on the size of the tire, and the staple shall be not less than 1⅛ inch long.

The compound from which the cushion is made shall be similar to the friction compound. Its thickness and width vary with the size of the tire.

The tread must contain at least 70 per cent. by volume of the best quality new wild or plantation rubber. The organic acetone extract shall not exceed 10.0 per cent., and the total sulphur 9.0 per cent. of the weight of the rubber as compounded. The tensile strength shall be at least 2400 pounds per square inch; the ultimate elongation 500 per cent.; the set, 10 minutes after it has been stretched 400 per cent. for 10 minutes, not more than 25 per cent. Its thickness over all and exclusive of non-skid portion is specified for the different sizes of tires.

The sidewall shall contain at least 65 per cent. by volume of the best quality, new wild or plantation rubber. The organic acetone extract shall not exceed 11.0 per cent., and the total sulphur 9.0 per cent. of the weight of the rubber as compounded. The tensile strength shall be at least 1600 pounds per square inch for all sizes up through 5 inch, and 2000 pounds for the larger sizes; the ultimate elongation 500 per cent.; and the set, 10 minutes after a stretch of 400 per cent. for 10 minutes, not more than 25 per cent. Its thickness varies with the size of the tire.

The bead construction of the different sizes of tires is governed by the reinforcing strips, the weight of these strips and finally by a hydraulic pressure test.

The tire sizes shall conform to the standards established by the Tire Division of the Rubber Association of America. The tires shall fit rims made according to the Tire and Rim Association standards.

Fabric tires under normal conditions of wear and stated S. A. E. loads shall give a mileage of at least 6000, and cord tires 8000.

The requirements for solid tires are briefly: The tread shall be made from a compound containing at least 65 per cent. by volume of the best quality new wild or plantation rubber. The organic acetone extract shall not exceed 10 per cent., and the total sulphur 8 per cent. of the rubber as compounded. Its tensile strength shall be not less than 2000 pounds per square inch; the ultimate elongation 450 per cent.; and the set not more than 40 per cent. 2 minutes after break. It shall be held securely to the base band by a hard rubber base. Other requirements consist of a rebound test, minimum area of cross section and a mileage guarantee of 10,000 under normal conditions of wear and stated S. A. E. loads.

The base bands shall be open hearth steel and conform to certain chemical requirements. They are to be electrically welded and shall give a minimum tensile strength of 45,000 pounds per square inch.

The inner tubes are divided into three classes: Pure gum, red antimony, and compounded tubes. The latter is to be used only in 6-inch or larger sizes.

The analytical and physical requirements are given in the table.

Analytical and Physical Requirements

Class	Rubber by volume Min.	‡Organic acetone extract Max.	‡Total sulphur Max.	(Gypsum) $CaSO_4.2H_2O$ Max.	Tensile strength Min.	Ultimate elongation Min.	Set stretch 1 to 6 inch Max.
	per cent.	per cent.	per cent.	per cent.	lb.-sq. in.	per cent.	per cent.
A.....	93	5.0	7.0		1800	750	8
B*....	85	5.0	7.0	7.0	2000	725	10
C†....	80	5.0	7.0		2500	600	15

*This class of tubes shall be compounded only of rubber and antimony sulphide.
†This class of tubes to be furnished only in 6 inch and larger sizes.
‡Based on the weight of rubber as compounded.

In addition the minimum diameter, length, thickness, volume of rubber, and an inflation test for the different sizes of tubes are specified.

Contraction and Expansions of Dental Amalgams.—The amalgams so extensively used in tooth filling are required to have a slight expansion on setting so as to fill the cavity tightly. If not sufficiently expanded the space will be quickly invaded by the fluids and organisms of the mouth, and decay will occur. If the expansion is too great the tooth may be broken. A. W. GRAY communicates to the *Physical Review* (1921, xviii, 108) the results of an extensive study of the properties of amalgams prepared from commercial alloys, and finds much difference in the different samples. The experiments were extended over considerable time. The general opinion is that tin causes contraction and silver expansion. A principal conclusion is as follows:

The contraction found when amalgam from a properly adjusted dental alloy is packed under a very high pressure does not in any way prevent the making of a tight tooth filling, because packing hard enough to cause contraction in such an amalgam will stretch the resilent dentin more than enough to make it follow the slight shrinkage of the filling. In fact, moderate contraction after very tight packing is an advantage, in that it relieves to some extent the straining of the tooth. The tighter the packing, the better the filling, because heavy packing pressure not only adds to the strength of the amalgam, but also shortens considerably the time required for it to complete all its dimensional changes and become stable. Moreover, it secures much better adaptation of the filling to the cavity walls and, consequently, reduces liability to leakage.

The phenomena described are to be expected from a consideration of conditions that influence diffusion, solution, and crystallization. Accurate measurements of the dimensional changes, termed *reaction expansions,* ought to throw light on other problems of metallography and physical chemistry.

H. L.

Production of Organic Compounds by Microörganisms.—In his presidental address entitled "The Laboratory of the Living Organism," DR. M. O. FORSTER, President of the Chemical Section, British Association for the Advancement of Science, pays tribute to the lowly yeast plant and its close relatives, certain bacteria and moulds. Yeast produces glycerol as well as alcohol. *Bacillus macerans* produces acetone and acetic and formic acids. The following organic compounds have also been obtained by the action of microörganisms: Acetaldehyde, dihydroxyacetone, butyl alcohol, butyric, oxalic, succinic, fumaric, lactic, citric, and pyruvic acids. If the proper genus and species of microörganism be chosen and be given the proper food and the proper environment, it will produce the desired organic compound, and will work 24 hours per day. Certain of these microbiological processes are used on a commercial scale, for instance in the manufacture of acetone and butyl alcohol. (*Scientific M.*, 1921, xiii, 301–308.)

J. S. H.

NOTES FROM THE RESEARCH LABORATORY EASTMAN KODAK COMPANY.*

ON THE RESTRAINT OF DEVELOPMENT BY BORAX AND CERTAIN SIMILAR SALTS.[1]

By E. R. Bullock.

It is well known that if borax, sodium bicarbonate or ordinary sodium phosphate be added to an ordinary alkaline developer, a restraining effect is produced. Upon repeating experiments on this in the laboratory, using lantern plates, the general results were confirmed. It was found, however, that the restraining effect of borax is more pronounced when the developer has been made up with a caustic alkali instead of with a carbonate and that with a bromided caustic soda hydroquinone developer, the addition of sodium carbonate causes acceleration; that of trisodium phosphate causes acceleration likewise, while sodium succinate has no appreciable effect. Lüppo-Cramer, who originally observed the restraining action of borax in development, concluded that it was due to a tendency to produce an insoluble silver salt, the effect not being given by sulphates, acetates, or tartrates, but only by those acids which form difficultly soluble silver salts, and he expressed the opinion that the principle involved is the same as in the case of restraint by soluble bromides.

The restraining effect of borax, however, and other similar acting salts when added to developers such as hydroquinone containing caustic soda is more easily explicable by the fact that these salts are those which while giving feebly alkaline solutions when dissolved in water, yet diminish the alkalinity of a more strongly alkaline solution. An example of such a salt is sodium bicarbonate. This when dissolved in water under the usual conditions gives a feebly alkaline solution, but when it is mixed with caustic soda, it combines with one chemical equivalent of the latter to form ordinary sodium carbonate, which is much less alkaline than the caustic soda itself, so that the addition of a weak alkali (sodium bicarbonate) to a strong alkali (caustic soda) produces a great

* Communicated by the Director.

[1] Communication No. 127 from the Research Laboratory of the Eastman Kodak Company.

diminution of alkalinity. The case of borax is more complicated in as much as several compounds intermediate in composition between it and caustic soda exist, but borax, although distinctly alkaline when dissolved alone in water, yet greatly diminishes the alkalinity of caustic soda.

The paradox of borax as an accelerator of development in certain cases and as a restrainer in other cases is thus entirely to be explained by changes in active alkalinity produced by the addition of borax to the developing solution. It may be mentioned that since borax has been recommended for use in developers to give a diminished grain that its effect is found to be due entirely to its alkalinity, and that the use of a corresponding developer made up to the same alkalinity with sodium carbonate produces the same effect.

Use of Perchloric Acid in Organic Analysis.—In the analysis of an organic compound, its nitrogen content is usually determined by the Kjeldahl method converting the nitrogen into ammonia by digestion with sulphuric acid, and titrating the ammonia formed. This method is almost universally used in the analysis of foods and of materials of biological origin. The digestion usually requires several hours, even though potassium sulphate be added to raise the boiling point of the sulphuric acid, and cupric sulphate be added as a catalyst. BRAINERD MEARS and ROBERT E. HUSSEY of Williams College (*Jour. Ind. Eng. Chem.*, 1921, xiii, 1054–1056) use 60 per cent. perchloric acid to hasten the digestion. This acid is added to the contents of the flask (sample, sulphuric acid, and cupric sulphate) prior to digestion. The amount of perchloric acid should be so governed that the contents of the flask become clear in not less than three minutes and not more than seven minutes after heating has begun. This result is usually attained if a one-gram sample, one gram of cupric sulphate, 25 c.c. of concentrated sulphuric acid, and 2 c.c. of perchloric acid be used. Digestion must be continued at least 15 minutes after the contents of the flask have become clear. Use of an excessive amount of perchloric acid gives rise to a loss of nitrogen. Distillation and titration of the ammonia are made in the usual way.

J. S. H.

Isomorphism of Sodium Sulphate and Sodium Chromate.—These salts, as dekahydrates, readily form mix-crystals. While the tetrahydrate of sodium sulphate does not exist, THEODORE W. RICHARDS and W. B. MELDRUM, of Harvard University (*Jour. Am. Chem. Soc.*, 1921, xliii, 1543–1545), have discovered mix-crystals of the tetrahydrates of sodium chromate and sodium sulphate. The tetrahydrated chromate crystals dissolve the sulphate as its tetrahydrate to form the mix-crystals. J. S. H.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

THE COMPOSITION OF CALIFORNIA LEMONS.[1]

By E. M. Chace, C. P. Wilson and C. G. Church.

[ABSTRACT.]

SAMPLES of the Eureka, Lisbon, and Villa Franca varieties of California lemons, from carefully selected trees located in the best lemon-growing districts of the State, were analyzed. The sampling was carried out at monthly intervals through a period of a year, with the following general results.

The differences in specific gravity of the fruit are the most striking of the few well-defined differences found to exist between the varieties examined. The Eureka variety has a greater specific gravity than either of the others; there is no apparent difference between the specific gravity of the Villa Franca and Lisbon varieties.

The Villa Francas contain more oil than the Eurekas. Otherwise, no absolute difference in the oil content is shown, although there is some indication that the Eureka contains less than the other varieties.

There is no difference in the acid content of the three varieties.

A marked difference in the sugar content of the Eureka and Lisbon varieties was shown.

California lemons have the lowest specific gravity during the winter months, and the highest in midsummer. The oil content is lowest in late winter and spring, and highest in the fall. The acidity is highest in the early fall months.

No correlation between the color of the peel and the composition of the fruit was found. As the thickness of the peel increases, the specific gravity of the fruit decreases, as does also the acid content.

No correlation between color and thickness of the peel was shown.

No difference in composition between lemons grown on the coast and those grown in inland valleys was detected.

* Communicated by the Chief of the Bureau.

[1] U. S. Department of Agriculture. *Bulletin 993*, issued October 15, 1921.

ALKALI FUSIONS. III—FUSION OF PHENYLGLYCINE-O-CARBOXYLIC ACID FOR THE PRODUCTION OF INDIGO.[2]

By Max Phillips.

[ABSTRACT.]

A STUDY of the fusion of phenylglycine-*o*-carboxylic acid with potassium hydroxide, sodium hydroxide, and various mixtures of these two alkalies has been made, and the optimum conditions for carrying out this reaction have been determined.

The results of the experiments with potassium hydroxide as the condensing agent indicate that 260° C. is the optimum fusion temperature, 10 minutes the optimum fusion period, and 12 to 16 moles potassium hydroxide to 1 mole phenylglycine-*o*-carboxylic acid, the optimum fusion mixture.

Using sodium hydroxide as the condensing agent, 270° C. has been found to be the optimum temperature, 25 to 30 minutes the optimum time, and 28 to 32 moles sodium hydroxide to 1 mole phenylglycine-*o*-carboxylic acid the optimum mixture.

Potassium hydroxide has been found to give higher yields of indigo than sodium hydroxide.

Mixtures of these two alkalies give better yields of indigo than sodium hydroxide alone, the yield increasing with the increase of the ratio of potassium hydroxide with respect to the sodium hydroxide.

DETERMINATION OF THE MONOAMINO ACIDS IN THE HYDROLYTIC CLEAVAGE PRODUCTS OF LACTALBUMIN.[3]

By D. Breese Jones and Carl O. Johns.

[ABSTRACT.]

THE lactalbumin was prepared from fresh skim milk. The casein was first precipitated at 35° C. by normal hydrochloric acid at a hydrogen ion concentration of approximately 4.6. The lactalbumin obtained by boiling the filtrate from the casein for 10 minutes was washed several times with hot water, and dried with alcohol and ether, and finally at 110° C.

The protein was hydrolyzed by boiling for 40 hours with hydrochloric acid (specific gravity 1.1) and the resulting monoamino acids determined, use being made of the more recent methods

[2] Published in *J. Ind. Eng. Chem.*, **13** (1921) : 759.

[3] Published in *J. Biol. Chem.*, **48** (1921) : 347.

for their separation. After saturating the hydrolysate with hydrochloric acid gas the greater part of the glutamic acid crystallized in the form of the hydrochloride. The diamino acids were then removed from the filtrate from the glutamic acid hydrochloride with phosphotungstic acid, and the solution of monoamino acids subjected to the lime-alcohol method for the separation of the remaining dicarboxylic acids. This removed aspartic acid, hydroxyglutamic acid, and the remainder of the glutamic acid. The monoamino acids recovered from the alcoholic filtrate from the calcium salts of the dicarboxylic acids were extracted with absolute alcohol for the removal of proline, and then esterified according to Foreman's lead salt method. The esters were fractionally distilled under diminished pressure and the amino acids recovered and isolated. Proline was determined in separate hydrolyses according to a method previously published by the authors. Serine was isolated from the unesterified fraction of the monoamino acids. The following percentages of amino acids were obtained: Glycine, 0.37; alanine, 2.41; valine, 3.30; leucine, 14.03; proline, 3.76; phenylalanine, 1.25; aspartic acid, 9.30; glutamic acid, 12.89; hydroxyglutamic acid, 10; serine, 1.76; tyrosine, 1.95.

The outstanding results of this analysis consist in the isolation of 0.37 per cent. of glycine, 1.75 per cent. of serine, and, at least, 10 per cent. of hydroxyglutamic acid. These amino acids have not been heretofore determined in the hydrolysis products of lactalbumin. A yield of 9.30 per cent. of aspartic acid was obtained, which is nine times the amount previously recorded. These percentages, together with those of the other amino acids, are tabulated and compared with the recorded results of a previous hydrolysis of lactalbumin by Abderhalden and Pribram.[4]

PINE-OIL AND PINE-DISTILLATE PRODUCT EMULSIONS: METHOD OF PRODUCTION, CHEMICAL PROPERTIES, AND DISINFECTANT ACTION.[5]

By L. P. Shippen and E. L. Griffin.

[ABSTRACT.]

AN investigation was undertaken to determine the physical, chemical and disinfectant properties of pine-oil and pine-distilla-

[4] *Zeit. Physiol. Chem.*, 51 (1907): 409.

[5] U. S. Department of Agriculture, *Bulletin 989*, issued October 9, 1921.

tion products. The following conclusions were reached as a result of this work:

Pine-oil emulsions made from steam-distilled pine oils, when freshly prepared, gave Hygienic Laboratory coefficients varying from 3.42 to 4.34, the average being 3.88. At the end of twelve months the average was 3.66.

A disinfectant prepared from destructive-distilled pine oil is weaker as well as more variable in its germicidal power against *B. typhosus* than is the Hygienic Laboratory pine-oil disinfectant. The samples examined gave Hygienic Laboratory coefficients of from 1.71 to 3.42.

Emulsions made from the other oils tested gave coefficients under 1. These preparations failed to emulsify completely in 10 per cent. concentration.

Pine-oil emulsions made from various grades of pine oils failed to kill *M. aureus* and *B. anthracis* in any dilution capable of emulsification.

In view of the results obtained these products should not be used for general disinfecting purposes.

When using pine-oil emulsions against *B. typhosus*, it is safer, for practical purposes, to employ a solution of five times the strength capable of killing the organism in five minutes. Thus a product showing by the Hygienic Laboratory method a killing power of $\frac{1}{500}$ should be used in a $\frac{1}{100}$, or 1 per cent., dilution. If a product will not give a dilution having a concentration five times that of the weakest concentration capable of killing *B. typhosus* in 15 minutes, and remain completely emulsified, it should not be used as a disinfectant.

Occurrence of the Aurora Line in the Spectrum of the Night Sky. LORD RAYLEIGH. (*Nature*, Oct. 13, 1921.)—From exposures made in Essex on 150 nights it is learned that this line can be photographed on two nights out of three without regard to the weather. Strangely enough, twenty-six exposures, made in Northumberland, three degrees nearer to the north pole, gave not a trace of the line, though the same instrument was used in both cases. Seasonal variation cannot be held responsible for the difference since observations at the two stations were made alternately. When a plate was exposed for five nights at the higher latitude the auroral line manifested itself.

G. F. S.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting held Wednesday, November 16, 1921.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, November 16, 1921.

PRESIDENT DR. WALTON CLARK *in the Chair.*

Additions to membership, since last report, 4.

A report of progress was presented by the Committee on Library.

The Chairman announced that the next business of the meeting would be the presentation of medals and certificates to the gentlemen whose inventions had been examined by the Committee on Science and the Arts and found worthy of recognition by the Institute. He then recognized Mr. W. Chattin Wetherill of the Science and the Arts Committee who introduced His Excellency, James Hartness, Governor of Vermont, recently awarded the Edward Longstreth Medal for his screw thread comparator. Mr. Wetherill said:

Mr. President: It is my very great privilege to present to you His Excellency, James Hartness, Governor of Vermont, the inventor of the Hartness Screw Thread Comparator.

This invention has to deal with the accurate measurement of threaded parts, and when it is realized that by means of threaded parts practically all machinery is held together, any device which can contribute to the accuracy of fit of such machine elements marks a step forward in the safety, dependability and service of machinery in general. The Hartness Screw Thread Comparator is a device by means of which a projection lantern is used to throw an enlarged image of the profile of a screw thread upon a screen. Upon this screen there is a drawing of two perfect screw thread profiles differing from each other by an amount equal to the allowable tolerance. It is, therefore, possible to see at a glance whether the projected profile comes within this tolerance or not. By contrasting this extremely simple method of measurement with the existing methods of gauging and measuring diameters by means of micrometer calipers, it is readily appreciated that this device marks a tremendous step forward.

In consideration of the ingenuity and novelty displayed in the design of this device, and in consideration of its successful development into commercial form, adding as it does another instrument of great value to the mechanic arts, The Franklin Institute, acting through its Committee on Science and the Arts, awards its Edward Longstreth Medal to His Excellency, James Hartness, of Springfield, Vermont, for his Screw Thread Comparator.

The Chairman then presented the Medal and accompanying documents to Governor Hartness who, in acknowledging this recognition, said:

Mr. President, Fellow Members of The Franklin Institute, Ladies and Gentlemen: One of the highest honors that can come to a worker in any field

is a sincere expression of praise by one fully competent. The awards issued by this Institute are of highest significance especially to one who has known the wonderful part the members of the Institute have played in bringing forward this age of Science and highly developed mechanism. It is with a deep sense of appreciation that I accept the Edward Longstreth Medal for invention presented to me by The Franklin Institute. It constitutes at once a token of commendation and an inspiration to further endeavor.

Dr. George A. Hoadley was then recognized and presented Mr. Thomas Willing Hicks of Minneapolis, Minnesota, who had been awarded the Edward Longstreth Medal for his "Once-over" Tiller. Dr. Hoadley said:

Mr. President: While the Committee on Science and the Arts is pleased to recommend the inventor of any useful mechanical device for recognition, it takes especial pleasure in presenting to you the inventor of a device having for its object an increased production from the soil, thus adding to our food supply. The fact that there is a direct relation between the thoroughness with which the soil is prepared for the seed and the quality and quantity of the resulting crop, led the inventor of the "Once-over" Tiller to devise a machine that in a single trip over the land would put it in the best possible condition for production.

Taking the most modern form of plow as a basis he added to it a vertical axis, supplied with steel cutting blades, fixed in such a position that the furrow from the plow would be thrown against these blades. Since the shaft upon which they are mounted rotates at a high rate of speed, the result is that the entire furrow is finely pulverized and ready to feed the rootlets of the growing plant.

In consideration of the invention and development to commercial form of a new and exceedingly useful type of machine for the preparation of a seedbed for grain or other seeds, The Franklin Institute, acting through its Committee on Science and the Arts, awards its Edward Longstreth Medal to Thomas Willing Hicks, Esquire, of Minneapolis, Minnesota, for his "Once-over" Tiller.

I take pleasure, Mr. President, in presenting Mr. Hicks, inventor of the "Once-over" Tiller.

The Chairman then presented the Medal and accompanying documents to Mr. Hicks, who, in acknowledging this recognition, said:

I am entirely mindful of the great honor which it is my mission here to receive. But I would make it plain that this token of your appreciation is accepted conditionally—that is to say—only will I permit myself to accept the Edward Longstreth Medal after admitting to you and myself the conviction that I was but the channel for the Thought—rather than the creator or originator of it. There is but one creator. I, of myself, gave nothing.

One would be dense, indeed, who did not sense the fact that the world is just now passing through a crucial hour. The bravest are appalled—and inspired—by the present situation and I wonder if you gentlemen of this Franklin Institute fully appreciate the extent to which you are a vital factor in helping to solve the problems now confronting us.

The Franklin Institute of the State of Pennsylvania, by rewarding, recognizing, encouraging and facilitating the art of discovery and invention, makes

it possible for those of us so endowed to manifest new inventions, ideas, plans and methods that, in the end, make us something better than ourselves. I disagree with Cowper; man's first invention was not a stool, but steps—steps hewn in the face of a cliff—and how symbolic is this of our present efforts to mount ever higher.

When Man took the hand of humanity away from the work of Production and lent it, for five long years, to the crime of Destruction—while at the same time increasing Consumption—the world was brought face to face with a condition that called for radical thought and action. The fact that every morning the whole human race wakes up hungry, and the necessity of the farmer producing more food in less time and with fewer facilities—this was the crucible out of which sprang the invention of the "Once-over" Tiller, and for which I am being honored to-night.

We of the great West look to you here of the East, and to you of the New England States to lead the way. Out in our great West country where "main streets" flourish, it is but natural that we turn to you of the older settlements for our examples. We look to the eastern states for initiative and we turn to the New England States for inspiration. We ask to be allowed to continue to hold you up as our ideals.

So, if, in coming down here to receive an honor, I, at the same time bring you a message from "main street" land, let it be this—"Lead on, you of the East and we of the great West will continue to follow; only asking that you continue to lead us ever up."

The President then recognized Mr. L. S. Storrs, President of the Connecticut Company, New Haven, Conn., who in presenting a beautifully constructed working model [illegible] of the original Van Depoele motor said:

The motor of which this is a [illegible] was used for hauling freight in the lower Naugatuck Valley, around Ansonia and Derby, to the, so-called, Naugatuck docks connecting with the Naugatuck Valley Steamboat Company which operated a line of steamers between New York and Naugatuck.

The Corporation operating this was known as the Ansonia, Derby and Birmingham Electric Line and was the outgrowth of the original Derby Horse Railway Company.

The original charter of the Derby Horse Railway Company was amended by the Legislature so as to allow the "use of any power except steam," as Mr. John B. Wallace, afterwards the first President of the company, expressed the opinion that it was practicable to operate street cars by electricity.

Definite steps towards actively commencing construction were taken in 1887, $25,000 bonds having been issued.

As appears from its official records, the Derby Company decided to directly purchase its material and equipment and do its own construction work, but the company arranged with a contractor for the entire construction and equipment of its road; who accepted as his compensation, the greater part or all of that company's securities, as such was then common practice in connection with the construction of horse railways in smaller communities.

At the first meeting of the directors held on April 13, 1887, the following resolution was adopted:

"*Resolved*, That the President be authorized to negotiate and make such contract with any company or companies controlling an electrical plan for operating street railways, for the electrical equipment of the road, as is in his opinion most advantageous for the Company."

And a contract was made with the Van Depoele Electric Company in April, 1887 which company for a sum of $20,115 agreed to completely equip the road electrically including setting of poles and stringing wires.

The patents of the company covered "generating and distributing of the current, the transmission and controlling of the same, the constructing of the motors and the various apparatus and devices used in all branches of the electric railway system."

This freight motor arrived in Derby on April 4, 1888 and was described as follows in *The Electrical Review* of April 21, 1888:

"ELECTRIC POWER

"The large electric motor to be used in Ansonia, Connecticut, on the electric road in drawing freight cars has arrived at the station there. It weighs several tons and is enclosed in a neat looking car about eighteen feet in length. Cars can be coupled on at either end and it is arranged to run forwards or backwards. One peculiarity is that the car in which it is enclosed sets down within six inches of the track. It was made at the works of the Pullman Car Company at Pullman, Illinois."

The track and overhead were ready for operation May 1, 1888, and on that date the line was opened, but not with the big motor. Some one had blundered, the roof of the car was too high to let it go under a low bridge near its objective, the steamboat dock. The necessary changes were made at once, the roof dropped two feet; the frame carrying the machine for the fringed and braced so it could be laid flat on the roof, and to give the operator room. The floor at each side of the motor was dropped and on July 1st the car went into operation.

The motor weighed 17½ tons and was capable of hauling 35 tons of freight.

One or two operating details may be of interest. The commutator bars had fibre separators, at the ends, but under the brushes there were air spaces between. The brushes of laminated copper, covered four bars. Under heavy load there was much arcing which fused the ends of the brushes; and when the car stopped suddenly the armature was apt to turn back a little due to play in the chain, and catching the ends of the brush in the spaces between the commutator bars, crumpled up the leaves. To remedy these troubles the motorman had a hammer and a pair of tinsmith's snips, with which he dressed the brush back into shape.

The motor was in operation up to December, 1888, when the river froze and stopped navigation for these months; on the opening of the river it again went into service, but in October, 1889, the Naugatuck Valley Steamboat Company went out of business and the motor was permanently retired. The passenger cars, however, continued to operate until replaced by later types.

On motion, duly seconded, the thanks of the Institute were extended to Mr. Storrs for his valuable gift.

The paper of the evening was presented by Dr. Heber D. Curtis, Director of the Allegheny Observatory, Pittsburgh, Pennsylvania, on "The Spiral Nebu-

læ and Their Interpretation." The speaker pointed out that of all known classes of celestial objects, the spirals are perhaps the most difficult to fit into any coherent scheme of stellar evolution, either as a point of origin or as an evolutionary product. In form, in distribution, and in space velocity, they stand apart from all other objects observed in the Milky Way. Much modern observational evidence supports the belief that these beautiful objects are separate galaxies of stars, or "island universes," comparable with our own galaxy in size and in number of component suns. Evidence for and against the island universe theory of the spirals was presented. The subject was illustrated by lantern slides.

After a brief discussion the unanimous thanks of the meeting were extended to Doctor Curtis for his interesting communication.

Adjourned.

R. B. OWENS,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of the Stated Meeting Held Wednesday, November 2, 1921.)

HALL OF THE INSTITUTE,
PHILADELPHIA, November 2, 1921.

DR. H. J. M. CREIGHTON, *in the Chair.*

The following reports were presented for first reading:

No. 2774. Cipherwriting Typewriter.
No. 2777. Chlorine Distributing Apparatus.

R. B. OWENS,
Secretary.

SECTIONS.

Section of Physics and Chemistry.—A meeting of the Section was held in the Hall of the Institute on Thursday evening, October 6, 1921, at 8 o'clock. Dr. Gellert Alleman in the Chair. The minutes of the previous meeting were approved as published.

Joseph S. Ames, Ph.D., LL.D., Professor of Physics in the Johns Hopkins University and Director of the Office of Aeronautical Intelligence of the National Advisory Committee for Aeronautics, delivered an address on "Aeronautic Research." The three types of aircraft—airplane, airship, and helicopter—were described. Scientific problems, which arise in connection with the various types, were mentioned; and the mode of solution was outlined. An account was given of wind-tunnel experiments and actual flying tests. The lecture was illustrated with lantern slides.

The paper was discussed. On motion of Prof. Monroe B. Snyder, a rising vote of thanks was extended to Doctor Ames. The meeting then adjourned.

JOSEPH S. HEPBURN,
Secretary.

Section of Physics and Chemistry.—A meeting of the Section was held in the Hall of the Institute on Thursday evening, October 13, 1921, at 8 o'clock. Dr. H. J. M. Creighton in the Chair. The minutes of the previous meeting were approved as read.

William Duane, A.M., Ph.D., Professor of Biophysics in Harvard University, presented a communication on "X-ray Spectra and Crystal Structure." The methods for the measurement of X-ray spectra with precision were described. The general character of, and the experimental relations between, continuous, line and X-ray absorption spectra were discussed. An account was given of the evidence obtained through the X-rays concerning the structure of crystals and of atoms, the mechanism of radiation, the quantum law, and the theory of relativity. In conclusion, stress was placed on the importance of spectrum analysis in medical applications of the X-rays.

The lecture was illustrated with lantern slides. After a discussion of the paper, a vote of thanks was extended to Doctor Duane; and the meeting adjourned.

JOSEPH S. HEPBURN,
Secretary.

Section of Physics and Chemistry.—A meeting of the Section was held in the Hall of the Institute on Thursday evening, October 27, 1921, at 8 o'clock. Dr. George A. Hoadley in the Chair. The minutes of the previous meeting were read and approved.

Henry Leffmann, A.M., M.D., Ph.D., D.D.S., Lecturer on Research in the Philadelphia College of Pharmacy and Science, delivered an address on "The Application of the Microscope to Research." An account was given of the use of color screens, polarized light, photography with ordinary and special plates, and the staining of minute objects. The lecture was illustrated with lantern slides. The paper was discussed at length; a vote of thanks was tendered Doctor Leffmann; and the meeting adjourned.

JOSEPH S. HEPBURN,
Secretary.

Electrical Section.—A meeting of the Section was held in the Hall of the Institute on Thursday evening, November 3, 1921, at 8 o'clock, with Mr. W. C. L. Eglin in the Chair. The minutes of the previous meeting were approved as published.

Fred. H. Albee, A.M., Sc.D., M.D., of New York City, delivered a lecture upon "The Use of Electric Machine Tools in Bone Surgery." The evolution of surgical instruments for the cutting of bone was traced; and an account was given of the construction and use of the automatic machine tools devised by Doctor Albee for use in bone surgery. The tools were exhibited; and lantern slides and moving pictures were used to illustrate the lectures. Doctors Skillern, Young, Albee and others participated in the discussion.

On motion of Doctor Snyder, a rising vote of thanks was extended to Doctor Albee. The meeting then adjourned.

JOSEPH S. HEPBURN,
Acting Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, November 9, 1921.)

ASSOCIATE.

MR. JAMES STOKLEY, JR., University of Pennsylvania, Philadelphia, Pennsylvania.

NON-RESIDENT.

DR. HARRY S. HOWER, 5709 Solway Street, Pittsburgh, Pennsylvania.
MR. N. M. LOWER, 292 Balph Avenue, Bellevue, Pennsylvania.
MR. EDWARD R. WEIDLEIN, Director of Mellon Institute of Industrial Research, Pittsburgh, Pennsylvania.

CHANGES OF ADDRESS.

MR. J. HOWARD CHAMBERS, 52nd and Media Streets, Philadelphia, Pennsylvania.
MR. F. H. CLARK, 1014 Munsey Building, Baltimore, Maryland.
DR. GEORGE L. KELLEY, 6804 McCallum Street, Philadelphia, Pennsylvania.
MR. HENRY HOBART KNOX, 160 Broadway, New York City, New York.
MR. CHARLES LONGSTRETH, Box 326, Coronado, California.
MR. RICHARD C. MCCALL, 2115 Locust Street, Philadelphia, Pennsylvania.
DR. C. O. MAILLOUX, 111 Fifth Avenue, New York City, New York.
DR. EDWARD MARBAKER, National Lamp Works, E. 45th Street and Hough Avenue, Cleveland, Ohio.
DR. P. G. NUTTING, 109 South Church Street, Schenectady, New York.
MR. JOHN B. RUMBOUGH, P. O. Box No. 2, Ashville, North Carolina.
MR. CHARLES W. SOULAS, 2022 North Park Avenue, Philadelphia, Pennsylvania.
MR. W. E. SYMONS, 108 West 43rd Street, New York City, New York.
MR. EARL H. TSCHUDY, 138 South Fourth Street, Lebanon, Pennsylvania.
MR. J. STEPH. VAN DER LINGEN, Department of Pharmacology, Johns Hopkins Medical School, Baltimore, Maryland.
MR. F. R. WADLEIGH, Fuel Division, Bureau of Foreign and Domestic Commerce, Department of Commerce, Washington, District of Columbia.

NECROLOGY.

Joseph William Richards, was born at Oldbury, England, on July 28, 1864, and died at Bethlehem, Pennsylvania, on October 12, 1921. While still a youth he came to the United States with his parents and was educated in the public schools of Philadelphia. He entered Lehigh University in 1886, and completed his education at the University of Heidelberg and the Mining Academy at Freiberg. From 1887 to 1897 he was instructor in Metallurgy and Mineralogy at Lehigh University, later he became asssistant professor, then acting professor and since 1903 has been professor of Metallurgy and Mineralogy in the same institution.

Doctor Richards acted as legal expert in many chemical and metallurgical litigations and was a member of the U. S. Assay Commission in 1897; a member of the jury of awards, and Chairman, Metallurgical Sub-jury Panama

Pacific Exposition, 1915; member, U. S. Navy Consulting Board, 1915-18; a member of the American Electrochemical Society, its first president, 1902, and secretary since 1907.

He was the author of the first English work on aluminum, published in 1887, and numerous other books on metallurgical subjects and a frequent contributor to the technical journals. Doctor Richards was a member of the leading technical societies of the world and became a member of The Franklin Institute on February 2, 1891.

Alexander Gray, Director of the School of Electrical Engineering at Cornell University, and well known as a writer and authority on electrical machinery, died on October 13, 1921, after a lingering illness.

Professor Gray was born in Edinburgh, Scotland, on March 9, 1882. While serving his apprenticeship he attended evening classes at Heriot-Watt College and graduated from Edinburgh University in civil and mechanical engineering in 1903. Awarded a Whitworth Scholarship, he then spent two years at McGill University, Montreal, in the study of electrical engineering.

After graduating from McGill University, Professor Gray joined the engineering staff of the Bullock Electric Company, of Cincinnati, and two years later transferred to the staff of the Allis-Chalmers Company at Milwaukee. During his five years of service with these companies he gained a wide experience in the design, construction and opration of all types of electrical machinery. In 1910 he accepted an appointment as assistant professor of electrical engineering at McGill University. In 1915 he was chosen to fill the position of head of the electrical engineering department in Sibley College, Cornell University. Upon reorganization of the college of engineering at Cornell in 1920, Professor Gray was made Director of the School of Electrical Engineering. A man of fine personality, a charming companion and an exceptionally strong teacher, from the very beginning of his professorship he made a warm place for himself with both students and faculty.

Professor Gray had been granted a considerable number of patents for electrical apparatus. He also contributed largely to the literature of his subject, and his books on electrical machine design, and on the principles and practice of electrical engineering are favorably known to the profession. In 1918 he was the recipient of the Howard N. Potts Medal of The Franklin Institute for his paper on "Modern Dynamo Electric Machinery." Professor Gray was also a member of numerous educational and scientific societies; he took an especially active interest in the work of the American Institute of Electrical Engineers, and served on a number of its technical committees.

In his death, the profession of electrical engineering loses a brilliant engineer and a great writer, and the teaching profession a teacher of rare activity.

Mr. George R. Henderson, Aldine Hotel, Philadelphia, Pennsylvania.

Mr. C. J. Hexamer, The Bartram, Philadelphia, Pennsylvania.

MINUTE OF THE BOARD OF MANAGERS RELATIVE TO THE DEATH OF MR. GEORGE R. HENDERSON.

At the meeting of the Board of Managers, held Wednesday, November 9, 1921, Mr. Coleman Sellers, Jr., addressing the President, said:

It is with sincere regret that the Managers of The Franklin Institute find it necessary to record the death on October 19th last of their fellow member, George R. Henderson.

Mr. Henderson joined the Institute February 23, 1912, and soon connected himself with the Committee on Science and the Arts. During 1914-15, he acted as Chairman of the Committee.

He was elected a manager in 1915, and served continuously from that date until his death.

Mr. Henderson was trained on the Pennsylvania Railroad at Altoona. Then went to the Norfolk and Western Railroad as Assistant to the Superintendent of Motive Power at Roanoke, Virginia. He was promoted to Mechanical Engineer and subsequently appointed Superintendent of Motive Power of the Atchison, Topeka and Santa Fe Railroad. He resigned this position and took up practice as a Consulting Engineer, specializing in railroad work, with offices in New York. He had some very important commissions, including the design and equipment of the railroad shops at Parsons, for the Missouri, Kansas and Texas Railway. He subsequently removed to Philadelphia and accepted a position with the Baldwin Locomotive Works, as Mechanical Engineer. This he retained as long as his physical condition permitted, when he retired from active work.

Mr. Henderson was a mechanical engineer of ability and wide experience, especially in railroad work. He was a well-informed man with a decided interest in science in general, especially in physics and astronomy. Personally, he was cordial, kindly and sociable. He was an attentive and efficient manager and his absence from our meetings will be felt, and his loss sincerely regretted.

The Secretary is requested to convey to Mrs. Henderson an expression of the sincere sympathy of the Managers in her great bereavement.

LIBRARY NOTES.

PURCHASES.

BANCROFT, WILDER D.—Applied Colloid Chemistry; General Theory. 1921.

BARNETT, E. DE BARRY.—Anthracene and Anthraquinone. 1921.

BLYTH, ALEX. WYNTER AND BLYTH, M. W.—Poisons: Their Effects and Detection. 1920.

FIERZ-DAVID, HANS EDOUARD.—Fundamental Processes of Dye Chemistry. 1921.

FISCHER, MARTIN H.—Soaps and Proteins. 1921.

FRIES, AMOS A. AND WEST, CLARENCE J.—Chemical Warfare. 1921.

HALL, WM. T. AND WILLIAMS, R. S.—Chemical and Metallographic Examination of Iron, Steel, and Brass. 1921.

MORECROFT, J. H.—Principles of Radio Communication. 1921.

PRING, J. N.—The Electric Furnace. 1921.

SMITH, ARTHUR BESSEY AND CAMPBELL, WILSON LEE.—Automatic Telephony. 1921.

SNELL, F. D.—Colorimetric Analysis. 1921.

SNELL, JOHN F. C.—Power House Design. 1921.

SVEDBERG, THE.—The Formation of Colloids. 1921.

WHITEHEAD, S. E.—Benzol: Its Recovery, Rectification and Uses. 1920.

GIFTS.

Air Brake Association, Questions and Answers on the U. C. Equipment. New York City, New York, no date. (From the Association.)

American District Steam Company, Bulletins 155 and 156 and Catalogue No. 21. North Tonawanda, New York, no date. (From the Company.)

American Engineering Company, The Central Station and the Taylor Stoker. Philadelphia, Pennsylvania, 1921. (From the Company.)

American Foundry Equipment Company, American Sand Blast Equipment. New York City, New York, no date. (From the Company.)

American Foundrymen's Association, Year Book for 1921. Chicago, Illinois. (From the Association.)

American Institute of Mining and Metallurgical Engineers, Transactions, vol. lxiv. New York City, New York, 1921. (From the Institute.)

American Iron and Steel Institute, Annual Statistical Report for 1920. New York City, New York, 1921. (From the Institute.)

American LaFrance Fire Engine Company, Inc., LaFrance Safety Devices. Elmira, New York, no date. (From the Company.)

American Wood-preservers' Association, Proceedings of the Seventh Annual Meeting. San Francisco, California, January, 1921. (From the Association.)

American Woolen Company, American Woolen Company Mills, Library edition. Boston, Massachusetts, 1921. (From the Company.)

Ashland Fire Brick Company, Refractories for Oil Burning Furnaces. Ashland, Kentucky, no date. (From the Company.)

Ashworth and Parker, La Machine a Vapeur Parker. Bury, England, no date. (From the Company.)

Atlas Car and Manufacturing Company, Catalogue on Cars, Electric Locomotives and Complete Industrial Railway Equipment. Cleveland, Ohio, no date. (From the Company.)

Austin Manufacturing Company, Catalogue G on The Austin Motor Sweeper. Chicago, Illinois, no date. (From the Company.)

Baker, R. and L., Company, Catalogue No. 11 on Industrial Tractors and Trucks. Cleveland, Ohio, no date. (From the Company.)

Bastian-Blessing Company, Catalogue No. 23 of Rego Welding and Cutting Apparatus. Chicago, Illinois, July, 1921. (From the Company.)

Bath, John, and Company, Inc., Bulletins 10, 15 and 20 of Micrometers. Worcester, Massachusetts, no date. (From the Company.)

Belliss and Morcom, Ltd., Fuel Economy. Birmingham, England, June, 1921. (From the Company.)

Blaw-Knox Company, Catalogue No. 23, Blawforms for General Concrete Construction. Pittsburgh, Pennsylvania, no date. (From the Company.)

Braby, Frederick, and Company, Ltd., Booklet on Braby Steel Structures and Handbook for Architects and Engineers. Glasgow, Scotland, no date. (From the Company.)

Brackett, F. W., and Company, Ltd., Air Compressors, Pumps, etc. Colchester, England, 1921. (From the Company.)

British Machine Tool Makers, Ltd., Machine Tools, First Edition. London, England, no date. (From the Company.)

British Scientific Instrument Research Association, Third Annual Report for the Year 1920-1921. London, England. (From the Secretary.)

Brown Brothers, Ltd., Catalogue of Electric Lighting and Starting Equipment for Motor Vehicles. Birmingham, England, no date. (From the Company.)

Brown Clutch Company, Hoists. Sandusky, Ohio, no date. (From the Company.)

Brown Hoisting Machinery Company, Catalogue K, Locomotive Cranes and Booklet on Man Power Multiplied. Cleveland, Ohio, 1921. (From the Company.)

Brown University, Catalogue 1920-1921. Providence, Rhode Island, December, 1920. (From the University.)

Browning Company, Browning Buckets. Cleveland, Ohio, 1918. (From the Company.)

Brunton's Alternating Stress Testing Machine. Musselburgh, Scotland, June, 1921. (From Brunton's.)

Buckton, Joshua, and Company, Ltd., Catalogue of Machine Tools and Testing Machines. Leeds, England, no date. (From the Company.)

Bucyrus Company, Hydraulic Dredges, Bulletin H-101. South Milwaukee, Wisconsin, no date. (From the Company.)

Bury Compressor Company, Bulletin No. 407 on Air Compressors. Erie, Pennsylvania, no date. (From the Company.)

Butters Brothers and Company, Catalogue No. 51 of Cranes. Glasgow, Scotland, no date. (From the Company.)

Calebough Self-Lubricating Carbon Company, Inc., Catalogue No. 6 of No-Spark Brushes. Philadelphia, Pennsylvania, 1921. (From the Company.)

Canada Grand Trunk Railway, Annual Report for 1920. Montreal, Canada. (From the Company.)

Chisholm-Moore Manufacturing Company, Catalogue 26 of Hoists. Cleveland, Ohio, no date. (From the Company.)

Cleveland, Cincinnati, Chicago and St. Louis Railway Company, 32nd Annual Report of the Board of Directors. Cincinnati, Ohio, 1920. (From the Directors.)

Cleveland Crane and Engineering Company, Catalogue No. 2 on Cranes. Wickliffe, Ohio, no date. (From the Company.)

College of William and Mary, Catalogue 1920-1921. Williamsburg, Virginia, April, 1921. (From the College.)

Columbia University, Catalogue 1920-1921. New York City, New York. (From the University.)

Conradson Machine Tool Company, Contract Work. Green Bay, Wisconsin, no date. (From the Company.)

Cutler-Hammer Manufacturing Company, Booklet on Elevator Controllers. Milwaukee, Wisconsin, no date. (From the Company.)

Dayton-Dowd, Bulletin 244 on Centrifugal Pumps. Quincy, Illinois, no date. (From the Company.)

DeLaval Separator Company, Catalogue No. 200 of DeLaval Method of Centrifugal Clarification of Varnishes, Japans and Pigment Goods. New York City, New York, no date. (From the Company.)

DeLaval Steam Turbine Company, Ten Years Progress in Water Works Pumps. Trenton, New Jersey, 1921. (From Dravo-Doyle Company.)

Detroit Stoker Company, Booklet on Detroit Stokers. Detroit, Michigan, no date. (From the Company.)

Diamant Tool and Manufacturing Company. Inc.. Circular on Diamant Dies. Newark, New Jersey, 1921. (From the Company.)

Dickinson College, Catalogue 1920-1921. Carlisle, Pennsylvania. (From the College.)

Dow, Charles Mason, Anthology and Bibliography of Niagara Falls. Vols. i and ii. Albany, New York, 1921. (From New York State Library.)

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Edwards, O. M., Catalogue H, Trap-Doors and Fixtures, and Catalogue W-19, Window Fixtures. Syracuse, New York, no date. (From the Company.)

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Farrell-Cheek Steel Foundry Company, The Illustrated Story of a Farrell-Cheek Steel Casting. Sandusky, Ohio, no date. (From the Company.)

Fisher Governor Company, Bulletin Catalogue on Power Plant Specialties and Catalogue T-19 on Steam Traps. Marshalltown, Iowa, no date. (From Sheffler-Gross Company, Philadelphia Representatives.)

Fry's Metal Foundry, Fry's White Metals. London, England, no date. (From the Company.)

Fuerst-Friedman Company, Price list of Motors. Cleveland, Ohio, no date. (From the Company.)

General Fireproofing Company, Saving with Shelving and Efficient Record Protection. Youngstown, Ohio, no date. (From the Company.)

Geometric Tool Company, Booklet on Geometric Screw Cutting Tools and Machines, fifth edition. No date. (From the Company.)

Georgetown University, General Catalogue, 1920-1921. Washington, District of Columbia. (From the University.)

Glover, W. T., and Company, Ltd., List No. 14, Glo-clad Electric Wiring System. Manchester, England, March, 1921. (From the Company.)

Good Roads Machinery Company, Inc., Champion Snow Plow. Kennett Square, Pennsylvania, no date. (From the Company.)

Goucher College, Catalogue for 1921-1922. Baltimore, Maryland. (From the College.)

Griscom-Russell Company, Bulletin No. 1140 on Stratton Steam Separators. Philadelphia, Pennsylvania, no date. (From the Company.)

Gwynnes. Ltd., Modern Centrifugal Pumping Machinery and Circulars Nos. 83, 84, L515, L518, L535, L539. London, England, no date. (From the Company.)

Hagan Corporation, Hagan De-Concentrator. Pittsburgh, Pennsylvania, no date. (From the Corporation.)

Hart Roller Bearing Company, Two booklets, Prepare for a Boom and Increasing Production. Orange, New Jersey, no date. (From the Company.)

Haverford College, Catalogue 1920-1921. Haverford, Pennsylvania, 1921. (From the College.)

Haverhill Engineering Department, Annual Report of the City Engineer. Haverhill, Massachusetts. 1921. (From the Department.)

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Heap, Joshua, and Company, Ltd., Catalogue of Screwing Machines and Tools. Ashton-under-Lyne, England, 1921. (From the Company.)

Holcroft's Steel Foundry Company, Ltd., Concerning Steel Castings. London, England, no date. (From the Company.)

Hunton, C. A., and Sons, Handy Machines and Twist Drills. London, England, no date. (From the Company.)

Hyatt Roller Bearing Company, Report on Trucks. New York City, New York, no date. (From the Company.)

Ideal Electric and Manufacturing Company, Bulletins 101 and 105 on Elevator Motors. Mansfield, Ohio, no date. (From the Company.)

Ingersoll-Rand Company, "Little David" Pneumatic Tool Accessories. New York City, New York, May, 1921. (From the Company.)

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International Railway Fuel Association, Proceedings of the Eighth, Ninth, Tenth and Eleventh Annual Conventions, 1916-1919. Chicago, Illinois. (From the Association.)

Insley Manufacturing Company, Catalogue No. 43. Indianapolis, Indiana, no date. (From the Company.)

Iron and Steel Institute, Journal, vol. ciii, No. 1, Charter, By-Laws and List of Members and Associates. London, England, 1921. (From the Secretary.)

Jeffrey Manufacturing Company, Catalogue No. 350 on Jeffrey Material Handling Machinery. Columbus, Ohio, no date. (From the Company.)

Jointless Fire Brick Company, Modern Furnace Building. Chicago, Illinois, 1921. (From the Company.)

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Lehigh University, Register, 1921. Bethlehem, Pennsylvania. (From the University.)

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Locomotive Stoker Company, The Duplex Locomotive Stoker. Pittsburgh, Pennsylvania, no date. (From the Company.)

Lonergan, J. E., Company, Catalogue No. 200 on Boiler, Steam and Gas Engine Fittings. Philadelphia, Pennsylvania, no date. (From the Company.)

Lussky, White and Coolidge, Inc., Catalogue E on Cabinet Hardware. Chicago, Illinois, 1918. (From the Company.)

Lynn Water Supply Commissioners, Report for 1920. Lynn, Massachusetts. (From the Commissioners.)

Manchester Association of Engineers, Transactions, 1919-1920. Manchester, England. (From the Secretary.)

Manley Manufacturing Company, Bulletins 61 and 62, Vises. York, Pennsylvania, no date. (From the Company.)

Massachusetts Institute of Technology, Catalogue on Courses of Study and Subjects of Instruction and General Information Requirements for Admission, First and Second Editions. Cambridge, Massachusetts, 1921. (From the Institute.)

Master Tool Company, Catalogue and Price List of Master Tool Reclaiming System, Their Parts and Appliances. Cleveland, Ohio, no date. (From the Company.)

Mechanical Appliance Company, Bulletins 401 to 405, inclusive, on Watson Motors. Milwaukee, Wisconsin, no date. (From the Company.)

Medford Water and Sewer Commissioners, Annual Report. Medford, Massachusetts, December, 1920. (From the Commissioners.)

Merchants Association of New York, Year Book, 1921. New York City, New York. (From the Association.)

Michigan Central Railroad Company, Seventy-fifth Annual Report. Detroit, Michigan, 1921. (From the Company.)

Michigan College of Mines, Year Book 1920-1921. Houghton, Michigan. (From the College.)

Modern Engine and Supply Company, Catalogue No. 21. Chicago, Illinois, 1921. (From the Company.)

Morse Dry Dock and Repair Company, Keeping the World's Ships in Good Condition. Brooklyn, New York, 1921. (From the Company.)

Nash Engineering Company, Bulletins Nos. 10, 11, 15, 16 and 17. South Norwalk, Connecticut, 1919. (From the Company.)

National Carbon Company, Inc., Catalogue No. 37 on National Carbon Brushes. Cleveland, Ohio, 1921. (From the Company.)

National Hoisting Engine Company, Catalogue on Hoisting Engines, Sixth Edition. Harrison, New Jersey, no date. (From the Company.)

National Malleable Castings Company, Booklet on Anchor Chain Cables. Cleveland, Ohio, no date. (From the Company.)

Nazel Engineering and Machine Works, Nazel Air Hammers. Philadelphia, Pennsylvania, 1921. (From the Works.)

Novo Engine Company, Catalogue No. 921. Lansing, Michigan, no date. (From the Company.)

Ohio State University, Catalogue 1920-1921. Columbus, Ohio. (From the University.)

Oilgear Company, Bulletin 4, Hydraulic Power Transmissions. Milwaukee, Wisconsin, no date. (From the Company.)

Oil, Paint and Drug Reporter, Green Book for Buyers, 1921 edition. New York City, New York. (From the Editor.)

Ontario Hydro-Electric Power Commission, Report on Hydro-Electric Power in the Niagara District. Montreal, Canada, no date. (From the Commission.)

Orton and Steinbrenner Company, O. S. Locomotive Cranes and Grab Buckets. Chicago, Illinois, no date. (From the Company.)

Pantasote Company, Catalogue of Waterproof Fiber Boards. New York City, New York, no date. (From the Company.)

Parson, C. A., and Company, Ltd., Pamphlet No. 3 on Steam Turbines to Paper Mills. Newcastle-on-Tyne, England, no date. (From the Company.)

Pease, C. F., Company, Catalogue of Blue Print Machinery and Drafting Room Supplies. Chicago, Illinois, no date. (From the Company.)

Pennsylvania State College, General Catalogues for 1919-1920 and 1920-1921. State College, Pennsylvania. (From the College.)

Pierce, Butler and Pierce Manufacturing Corporation, Booklet on Pierce Packless Valves. New York City, New York, July, 1921. (From the Corporation.)

Polytechnic Institute of Brooklyn, The Story of the Polytechnic and Its Service to the Public. New York City, New York, 1920. (From the Institute.)

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Rockford Twist Drill Company, Catalogue on Drills. Rockford, Illinois, no date. (From the Company.)

Ruson and Hornsby, Ltd., Oil Engines. London, England, no date. (From the Company.)

Russell and Stoll Company, Bulletin No. 50 on Wiring Fittings. New York City, New York, July, 1921. (From the Company.)

Ryerson, Joseph T., and Son, Ryerson Handbook on Alloy Steels. Chicago, Illinois, 1921. (From the Company.)

St. Louis Water Commissioners, Annual Report for year ending April, 1921. St. Louis, Missouri. (From the Commissioners.)

Sanderson Cyclone Drill Company, Big Blast Hole Drills. Orville, Ohio, 1921. (From the Company.)

Sharon Pressed Steel Company, Sharon Products. Sharon, Pennsylvania, no date. (From the Company.)

Shaw, Francis and Company, Ltd., Rubber and Allied Trades Machinery. London, England, no date. (From the Company.)

Simon-Carves, Ltd., Reinforced Concrete Engineers and Coal Washeries. Manchester, England, no date. (From the Company.)

Southern Railway Company, Twenty-seventh Annual Report. New York City, New York, 1921. (From the Company.)

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Structural Slate Company, Chapters 6, 7 and 8 of a series on Structural Slate. Pen Argyl, Pennsylvania, no date. (From the Company.)

Thermoid Ribber Company, Universal Joints, Their Use and Misuse. Trenton, New Jersey, 1919. (From the Company.)

Thoner and Martens, Catalogue of Disconnecting Switches, Heavy Duty Switches and T and M Seitch Locks. Boston, Massachusetts, no date. (From the Company.)

Thornycroft, John I., and Company, Ltd., Catalogue of Motor Vehicles. London, England, 1920. (From the Company.)

Thwing Instrument Company, Bulletins 10 and 17. Philadelphia, Pennsylvania, 1921. (From the Company.)

Tufts College, Annual Catalogue 1920-1921. Boston, Massachusetts. (From the College.)

Turton Brothers and Matthews, Ltd., Catalogue of Shear Blades, Dies, Punches, etc. Sheffield, England. (From the Company.)

Union University, Announcements from Medical Department, 1915-1921. Albany, New York. (From the University.)

Universal Winding Company, Electrical Coil Winding Machinery. Boston, Massachusetts, no date. (From the Company.)

University of Maryland, Bulletins Nos. 240 and 241. College Park, Maryland, 1920. (From the University.)

University of South Carolina, Catalogue 1920-1921. Columbia, South Carolina, April, 1921. (From the University.)

Ursinus College, Catalogues for 1919-1921. Collegeville, Pennsylvania. (From the College.)

Uskside Engineering Co., Ltd., Three Throw Ram Pumps, Haulage, Mining Plant, Hauling Engines. Newport, England, no date. (From the Company.)

Wallwork, Henry and Company, Ltd., Wallwork Worm Gear Drives. Manchester, England, no date. (From the Company.)

Warren-Knight Company, Sterling Transits and Levels. Philadelphia, Pennsylvania, no date. (From the Company.)

Watts, Fingham and Company, Ltd., Catalogue of Machinery, Tools, etc. London, England, no date. (From the Company.)

Wellesley College, Calendars 1916-1921. Wellesley, Massachusetts. (From the College.)

Wellman-Seaver-Morgan Company, Bulletins 63, 64 and 65 on Hydraulic Turbine Equipment. Cleveland, Ohio, July, 1921. (From the Company.)

Wesleyan University, Catalogues for 1917 to 1920. Middletown, Connecticut, 1917 to 1920. (From the University.)

Wheeler Condenser and Engineering Company, Wheeler Condensers. Carteret, New Jersey, 1921. (From the Company.)

Will Corporation, Abridged Catalogue of Laboratory Apparatus. Rochester, New York, no date. (From the Corporation.)

Williams Tool Corporation, Don't Let it Happen to You. Erie, Pennsylvania, no date. (From the Corporation.)

Wright, Joseph, and Company, Ltd., Booklet on Chains, Slings, Cables and Anchors. London, England, no date. (From the Company.)

BOOK NOTICES.

DIE BINOKULAREN INSTRUMENTE. By Moritz von Rohr. Second edition, with 136 figures, pp. xvii 303, one table. Berlin, Julius Springer, 1920.

This is an amplified and corrected second edition of v. Rohr's book of 1907. The Theoretical Part occupies but a small part of the volume, and deals with the essentials of the theory of binocular vision. The remainder of the work is dedicated to the history of binocular instruments together with the historical evolution of the concepts about the seeing with two eyes, from Euclid (300 B.C.) and the Greek philosopher Artemidor (100 B.C.) up to 1910. The chief publications on binocular vision which appeared since 1910 are tabulated at the end of the volume with reference numbers to the general historical index. The latter (pp. 262-271) will be particularly useful for those who contemplate further special historical studies of the subject.

LUDWIK SILBERSTEIN.

IMPERIAL INSTITUTE. Monographs on mineral resources with special reference to the British Empire prepared under the direction of the Mineral Resources Committee, of the Imperial Institute, with the assistance of the scientific and technical staff. Silver Ores, by H. B. Cronshaw, B.A., Ph.D., A.R.S.M. 152 pages, illustrations, 8vo. London, John Murray, 1921. Price 6 shillings, net.

This is one of a series on mineral resources with special reference to the British Empire. It is, as the title indicates, devoted entirely to silver ores. It contains a very large amount of information concerning the sources of silver, the amount mined and notes of the characters of the ores. The United States produced in 1916 nearly half the silver obtained in the world. A map on the Mercator projection shows the location of all the mines mentioned in the text. Many statistical tables are given, but there is also a large amount of descriptive matter which shows that the author, who was until lately a teacher of geology, has made careful and extensive studies of the regions covered by the essay.

HENRY LEFFMANN.

DIE TECHNIK DER ELEKTRISCHEN MESSGERÄTE. By Georg Keinath, Eng.D. 8vo., vii-443 pages, index and 372 illustrations in the text. R. Oldenbourg, Munich and Berlin. Paper-bound, 112 marks; cloth, 122 marks.

This work has grown out of a lecture delivered before the Electrotechnic Association of the Rhine-Westphalian area, in 1919, in connection with a high school course. The text of this lecture was published and with a view to extend the scope the present volume has been prepared. It covers the technical field only, inasmuch as the majority of available works in this field are not written

by persons actually engaged in the practical side, and, this side has been especially presented, since there are available good works on the side of purely scientific methods. Many forms of apparatus known only in the records of the patent offices have been left unnoticed, and more space has been given to description of the form and construction of the instruments, than to the theoretic questions and mathematical demonstrations. The large use of graphic methods will be acceptable to all users of the volume. The tenor of the descriptions leans somewhat towards the instruments about to come into use, and terms formed in agglutinative method so common to the German language are freely employed. This procedure is rather prominent now in Germany. A strong disposition is manifested to get rid of all terms having a foreign flavor. It is, in the main to be regretted, but it is probably an aftermath of war, and is, fortunately, less serious in the case of the German language because the individual words that are combined in the new terms are generally sufficiently suggestive to enable any one reasonably familiar with the language to work out the meaning. In photography, for instance, "Lichtbild Kunst" and "Lichtbild" have taken the place of "photographic art" and "photograph." In the work before us, we have such terms as "Dreheiseninstrumente," "Nebenwiderstand," "Vorwiderstand," which the author says need no explanation nor justification.

The illustrations are of very high quality. Many of them are from blocks furnished by the manufacturers of the instruments, and as the paper and presswork are both of good quality, the apparatus is shown in clear and distinct form. Apparatus from one firm is especially illustrated, a result, the author tells us largely due to his extensive use of such instruments.

It is not possible to summarize in the space allotted to a review the many excellencies of the book. It is, indeed, astonishing that a nation just emerging from a most disastrous war and burdened with both domestic and international complications, should be able to produce a volume of such an extent and thoroughness. It would seem to include about all that the technical electrical engineer will need in elucidating the construction, principles and operation of the measuring instruments used in the commercial applications of electricity. Incidentally, the author remarks that in the earlier instruments the form of the case was determined by the form of the machinery, but that now-a-days, the machinery is often adapted to the form of the case, a condition which is not always to the advantage of the instrument.

The fundamental considerations of exactness in electrical measuring instruments are, of course, much the same as in all other similar apparatus, but three principal errors are stated and given full consideration, namely: Inherent defects of the apparatus, defect in the method of measurement, errors in reading the results.

The book is commendable in all respects, and really one of the many instances of the extraordinary thoroughness, patience and purely scientific spirit in which the German carries out his task.

HENRY LEFFMANN.

NOS USINES MÉTALLURGIQUES DÉVASTÉES. (Our Devastated Metallurgical Plants.) This quarto volume of over 200 pages with many clear and vivid illustrations is published by La Revue de Métallurgie.

It shows the damage done deliberately to the many industrial plants in France and Belgium during the German occupation. The work has been prepared under the supervision of a group of engineers, an introduction being furnished by Prof. Leon Guillet of the Conservatory of Arts and Trades in Paris.

In the circular accompanying the copy, it is remarked that perhaps some will regard the publication as unnecessary at this late date, as the world has now other troubles and has no time to weep over the deeds of the war-days. The collection of photographs, however, is very instructive and informing as showing the character of modern war when carried out by an enemy that obeys no rule or agreement. The sight of these ruins is very impressive to the French and Belgian peoples and shows that their fear of future war is much more developed in regard to armies than in regard to navies, and explains the anxiety of France in regard to the results of the present conference on the limitation of armaments, which seems to be directing its attention almost entirely to the restriction of the sea-power.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 120. Practical Stability and Controllability of Airplanes, by F. H. Norton. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1921.

The effect of the characteristics of an airplane on balance, stability, and controllability, based on free flight tests, is discussed particularly in respect to the longitudinal motion. It is shown that the amount of longitudinal stability can be varied by changing the position of the center of gravity or by varying the aspect ratio of the tail plane, and that the stability for any particular air speed can be varied by changing the camber of the tail plane. It is found that complete longitudinal stability may be obtained even when the tail plane is at all times a lifting surface. Empirical values are given for the characteristics of a new airplane for producing any desired amount of stability and control, or to correct the faults of an airplane already constructed.

Report No. 122. Preliminary Experiments to Determine Scale and Slip Stream Effects on a 1/24th Size Model of a JN4H Biplane, by D. L. Bacon. 10 pages, illustrations, quarto. Washington, Government Printing Office, 1921.

This work was undertaken at the Langley Field Aerodynamic Laboratory to obtain results on a small model of a complete airplane which might be used for comparison with corresponding tests made in full flight. Somewhat similar tests have been previously made at various other laboratories; but as certain discrepancies exist between corresponding tests in different tunnels, it has been deemed advisable to obtain a direct comparison for this particular installation.

The present work covers tests on a 1/24th scale model at speeds varying from 6.7 m./sec. (15 m.p.h.) to 40.2 m./sec. (90 m.p.h.). A slip stream correction has been obtained by the use of a small belt-driven propeller mounted in front of the model, and force coefficients thus obtained are compared with the measurements of the same forces made in full flight on a geometrically similar airplane.

This report gives lift, drag, and longitudinal moment values obtained in tests of a particularly accurate model over a wide range of speeds. A measure

of the slip stream corrections on lift and drag forces was obtained by the use of power-driven model propeller.

Measurements were also made of forces and longitudinal moments for all angles from 0° to 360°.

PUBLICATIONS RECEIVED.

Eléments d'Analyse Mathématique a l'Usage des Candidats au Certificat de Mathématique Générales des Ingenieurs et de Physiciens. Cours Professé a l'Ecole Centrale des Arts et Manufactures, par Paul Appell. Fourth edition, entirely rewritten. 715 pages, illustrations, 8vo. Paris, Gauthier—Villars et Cie, 1921. Price 65 Francs.

Study Questions in Elementary Organic Chemistry, by Alexander Lowy, Ph.D., and Thomas B. Downey, M.S. 91 pages, 8vo. New York, D. Van Nostrand Company, 1921. [The study questions contained in this work are used in connection with the elementary organic chemistry course given at the University of Pittsburgh. They are based on the contents of a general elementary course. A list of reference books for collateral reading and study is given.]

Machine Drawing. A test and problem book for technical students and draftsmen by Carl L. Svenson, M.E., assistant professor of Engineering Drawing in the Ohio State University. 214 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1921.

U. S. Bureau of Mines. Monthly statement of Coal-mine Fatalities in the United States, August, 1921, by W. W. Adams. 10 pages, 8vo. Technical Paper 249, the Determination of Oxides of Nitrogen by V. C. Allison, W. L. Parker, and G. W. Jones. 13 pages, illustrations, 8vo. Technical Paper 291, Production of Explosives in the United States during the Calendar Year 1920, with notes on mine accidents due to explosives by William W. Adams. 44 pages, illustrations, 8vo. Technical Paper 293, Coke-oven Accidents in the United States during the Calendar Year 1920, by William W. Adams. 32 pages, 8vo. Washington, Government Printing Office 1921.

National Advisory Committee for Aeronautics: Technical Notes No. 65, Langley Field Wind Tunnel Apparatus by D. L. Bacon. 4 pages, illustrations, quarto. No. 71, Experiments with Slotted Wings, translated from "Zeitschrift für Flugtechnik und Motorluftschiffahrt," June 15, 1921. 32 pages illustrations, quarto. No. 72, Aneroid Investigations in Germany, prepared by M. D. Hersey. 9 pages, quarto. No. 73, The Choice of Wing Sections for Airplanes by Edward P. Warner, 30 pages, folded plates, quarto. Washington, Committee, 1921.

Exchange Stabilization. Hearings before the Committee on Banking and Currency of the House of Representatives on H.R. 8404 to investigate the international exchange problem for the purpose of determining the means which may best be employed for the stabilization of exchange, Saturday, October 8, 1921. Statement by Mr. H. N. Lawrie economist, American Mining Congress. 51 pages, tables, 8vo. Washington, Government Printing Office, 1921.

Absolute Sizes of Certain Monovalent Ions, by Wheeler P. Davey. 3 pages, 8vo. Reprinted from the *Physical Review*, N.S., volume xviii, No. 2, August, 1921.

CURRENT TOPICS.

The Limits of the Validity of the Geometrical Law Relating to the Refraction of Light. P. FROELICH. (*Ann. der Physik.*, No. 15, 1921.)—Let a ray of light travel through water upward toward air above the water. The geometrical law governing such a case predicts that this ray will emerge into the air or will be totally reflected back into the water according to the size of the angle of incidence. In fact, the matter is not so simple. A more complete theory indicates and experiment verifies that light really does penetrate into the air even when the geometrical law points to nothing but total reflection. The intensity of the light finding its way into the air diminishes rapidly with the depth of penetration.

Let us now consider the case of light originating in air and incident on a surface of water. One ray strikes the surface at right angles and enters with no change of direction. A second ray making an angle of 5° with the first meets the surface at a different point, suffers a change of direction and is continued by a ray of light in the water. A ray the direction of which in air makes 10° with the first encounters the surface of the water at a still different place, there experiences a change in direction greater than that of the second ray and in the water travels in a direction different from that followed in the water by the second ray. As rays farther and farther from the first are traced their points of contact with the water are progressively more distant, the changes of direction at the surface grow greater and greater and the refracted rays within the water make larger and larger angles with a perpendicular to the suface. When the ray in air just grazes the surface the refracted ray makes its maximum angle with the perpendicular. This angle is designated the critical angle.

Within the water let a straight tube be placed in the direction of the first refracted ray. Light from the source in air will come through it and an eye in the water would see the source through the tube. If the tube were shifted to one side and turned so as to lie along the second refracted ray the eye could again perceive the source. So long as the direction of the tube when prolonged to meet the surface makes an angle with the perpendicular there that is not greater than the critical angle it is possible by moving the tube about to find a position at which the source can be seen through it. If, however, the angle be made greater than the critical angle, there is no position at which the source can be seen through the tube. At least there should not be were the geometrical law strictly true, but again here theory and experiment unite to show that the angle may in reality be greater than the critical angle and still the source may be seen

through the tube, provided that the source be very close to the surface of separation.

The purpose of this investigation is to measure the maximum distance of the source from the surface for different departures of the line of sight beyond the critical angle. Glass was used instead of water. The distance from the source to the surface was measured by a method based on Newton's rings. One set of observations was made with green light of wave-length .000522 mm., for which the critical angle of the glass employed was 38.0°. To see the source in air along a line of sight making an angle with the perpendicular to the surface 2.5° greater than the critical angle, the source could not be farther from the surface than 2.51 wave-lengths or .001310 mm. For 40° beyond the critical angle the corresponding quantities are .613 wave-lengths or .000320 mm. With the same angles the distances when red light was used are all larger, though equivalent to a smaller number of wave-lengths.

G. F. S.

On the Variation of Resistance of Selenium with Temperature. SNEHAMOY DATTA. (*Phil. Mag.*, Sept., 1921.)—The curious sensitiveness of selenium to light, which shows itself by an enormous change in electrical resistance when the element is illuminated, has made this substance an important, even an essential, factor in transmitting light effects, such as pictures and photographs, to a distant point. The present investigator sets himself the task of finding how far the change of resistance under illumination is due to a change of temperature caused by the absorption of the incident radiation. The resistances of several selenium cells was measured from 0° C. to 170° C. Above the latter temperature sublimation begins. At 217° C. is the melting point. For one specimen the resistance at zero was 66.0×10^5 ohms, while at 170° C. it was a mere 2×10^5 ohms.

To determine how largely the change of temperature bulks in the change of resistance consequent upon illumination a cell was measured in the dark and its resistance was found to be 67.0×10^5 ohms. It was then exposed to light from which both ultra-violet and infra-red wave-lengths had been abstracted. The resistance was then 48.0×10^5 ohms, a decrease of 19×10^5 ohms. A thermo-couple showed that the selenium had risen about one degree. From the previous measurements this elevation of temperature would cause a fall in the resistance of $.75 \times 10^5$ ohms. It thus appears that the heating effect is competent to account for only about one twenty-fifth of the entire change in resistance.

Three different types of selenium crystals were obtained. One variety was "primarily non-conducting," a second was only slightly conducting and on this property change of temperature had no effect. The third type, appearing red in reflected light, was moderately conducting at the temperature of the room. Upon warming the red color gradually vanished and at the same time the resistance grew less. This type of crystal may, therefore, well be responsible for the

resistance change of selenium with rise of temperature. The color change of the crystals points to some sort of a transformation.

G. F. S.

The Age of the Earth. (*Nature*, Oct. 13, 1921.)—A discussion on this subject was held at the recent meeting of the British Association for the Advancement of Science at Edinburgh at which the points of view of physics, geology and astronomy were presented. Lord Rayleigh arrived at the age of uranium-bearing rocks from a study of their amounts of uranium-lead and helium. The rates of transformation from uranium through all the intermediate steps down to lead are known. This is not ordinary lead but lead of atomic weight 206. Assuming that there has been no change in the rates of transformation an age of 925,000,000 years is derived from these rocks. Considerations based on the helium content lead to concordant results. He assigns a period of a few billion years as the duration of the earth's crust in a condition fit for human habitation.

The geologists did not present an undivided front in respect to the proper interpretation of the data of their science. Professor Gregory held that prevalent age estimates might well be multiplied by ten or twenty. Results thus obtained would agree with those emanating from the field of physics.

Eddington from considerations based on certain variable stars concluded that Lord Kelvin's famous time-scale must be multiplied by 700. Jeffreys used two entirely distinct lines of approach, temperature distribution downward in the earth's crust and the evolution of the solar system by tides. In both cases he derives about two billion years as the length of time since the solidification of the crust.

G. F. S.

An Anticipation of One of the Results of Einstein's Theory of Relativity. P. LENARD. (*Ann. d. Physik*, Number 15, 1921.)—Einstein predicted that a ray of light from a star would be deviated from its path when it passes near the sun. This was confirmed by photographs of stars taken a couple of years ago at the time of a total eclipse of the sun. Professor Lenard, who is, to say the least, not strongly prejudiced in favor of the theory associated with the name of Einstein, takes pleasure in directing attention to a forgotten article, published by J. Soldner in 1801. The latter was a German mathematician, born in Bavaria, the son of a peasant. According to Professor Lenard's somewhat caustic comment, "He had besides the advantage of not having attended too many schools." In the article mentioned this scientist of Napoleonic times attributed to a ray of light all the absolute properties of matter and considered it as subject to gravitation. On this basis he calculated the deviation such a ray would experience in passing near the sun and arrived at an expression which Einstein likewise reached more than a century later from considerations of the Relativity Theory.

G. F. S.

Escapements and Quanta. SIR JOSEPH LARMOR. (*Phil. Mag.*, Oct., 1921.)—One outstanding difficulty in the Rutherfordian atom is to account for the emission of radiation from the outer ring of electrons, to explain the continuous emission of energy, if no energy is imparted to the emitting system. With that characteristically British longing for a concrete, material model—a bent that made Faraday regard his lines of force as real, elastic entities, that constrained Lord Kelvin to continue his search for a satisfactory type of ether, and that caused Sir Oliver Lodge to devise the ingenious and illuminating models of his "Modern Views of Electricity"—the Oxford don puts the arousing question "Might not an atom be a clock?" The pendulum of a clock keeps on swinging because at the right time and in the proper quantity it has its energy replenished from an accessible store. It is suggested that the outer electron-system corresponds to the pendulum, and that it is nourished by some kind of sublimated, dynamical escapement system from a stock of energy in the core of the atom. The bowed violin string and the blown organ pipe are invoked by way of illustration. "All which is parabolic, yet to a vivid imagination may be fertile in analogy." In relation to quanta the remark is made, "Wherever it proves necessary in physical science to treat of discrete quanta of energy, it may well be that these are packets separated in the cases concerned by the atomic mechanism —just as period in natural radiation is said in a certain sense to be a creation of the resolving prism or grating—without having to face the difficult assumption that energy is itself necessarily discrete. The quanta of practical physics would of course be large multiples of such packets."

G. F. S.

The Mental Multiplication and Division of Large Numbers. V. A. BAILEY, Queen's College, Oxford. (*Phil. Mag.*, Sept., 1921.)—The man on the street usually regards the methods employed by himself for the performance of the operations of multiplication and division of numbers as established from eternity and immutably derived from the constitution of the human mind. Perhaps more than one member of the A.E.F. was shocked out of his self-complacency to discover small French children doing long division by a way he knew not and by a method he could not at once follow, though he could see that it reduced the time requisite for the process.

The Oxford mathematician here presents a method of multiplication for which he advances the claims that it is speedier than the common methods, that it is less liable to error and that it is less fatiguing for large numbers. He makes very clear each step of the procedure. It is indeed surprising how simple the new method is to learn and how, after a few trials, it competes in quickness with the older method practised for decades.

G. F. S.

Corrosion of Metals in Alkaline Soils.—W. NELSON SMITH and J. W. SHIPLEY presented at the tenth general meeting of the Engineering Institute of Canada, Saskatoon, August 11, 1921, a paper giving an account of extended experiments on the corrosion of metal conduits by soil solutions. The paper appears in full in a recent issue of the *Engineering Institute's Journal*. The conclusions are:

1. The corrosion of cast iron by the salt solutions found in natural soil is readily accomplished under natural conditions without access of stray current. The corrosion is of the so-called graphitic pitting type by which is meant the commonly observed condition of the material remaining in place, which is invariably of a soft spongy texture with part of the iron dissolved out, the remainder resembling graphite in consistency.

2. Magnesium salts are the most corrosive of the soil salts, and magnesium sulphate, which was found wherever a cast iron pipe had been destroyed, is apparently the most effective of the salts experimented with.

3. Local action induced by naturally occurring concentration cells may easily be a factor in the pitting of cast iron exposed to salts of varying concentration.

4. Slight pitting corrosion was found in pieces of cast iron exposed to the action of small samples of wet soil and intermittently heated, even in the short period of forty days, and with only a limited supply of water as compared with conditions in natural ground, no impressed e.m.f. being present. H. L.

On the Velocity of Sound in Gases at High Temperature and the Ratio of Specific Heats. H. B. DIXON, COLIN CAMPBELL and A. PARKER. (*Proc. Royal Soc.*, A 702.)—The time required for a sound wave to traverse the gas contained in a tube of known length was measured by a pendulum chronograph. The material of the tube was selected so as to avoid chemical action between the contained gas and the tube. For air and carbon dioxide it was desirable to have a tube of fused silica. This was difficult to get since the length was to be 14 meters. The Thermal Syndicate of Newcastle-on-Tyne accomplished the task, and though the first two long tubes coiled into spirals broke in transit and the third broke upon heating the fourth proved satisfactory.

The velocity of nitrogen was measured up to 1000° C. for which temperature the value of 696.8 m. per sec. is given. At 0° C. it is 334.4. The velocity in air was determined up to 700° C., while 600° is the limit for carbon dioxide and methane. From the velocities in the tube the velocities in free air are calculated. The data for specific heats at constant pressure and at constant volume are also tabulated. The ratio of the former to the latter in the case of nitrogen sinks from 1.408 at 0° C. to 1.374 at 1000° C. G. F. S.

On the Electrical Conductivity of Some Dielectrics. H. H. POOLE. (*Phil. Mag.*, Oct., 1921.)—By means of a transformer and two thermionic rectifying valves the opposite, parallel surfaces of the dielectric were brought to a difference of potential. The current that then passed through the insulator was measured by the deflection of a galvanometer. From the potential difference per unit of thickness and the current the conductivity of the specimen was calculated. In the experiments the difference of potential was progressively raised until the dielectric was punctured. In the case of mica this took place when the potential gradient was about 2.9 megavolts per centimeter. For glass the corresponding number is .6; for paraffin wax, .25; for shellac, from .04 to .25; and for celluloid, .16.

When a conductor of electricity, such as copper, is investigated by the method sketched above it is found that the current flowing through it varies directly as the applied difference of potential. This relation is what is expressed in Ohm's Law. In other words, the conductivity of copper is independent of the current flowing through it and of the difference of potential applied, so long as all other physical properties undergo no change. With dielectrics quite a different relation holds, as Mr. Poole brought out in papers published several years ago. He finds that the higher the applied potential difference the greater is the conductivity. The relation is expressed by the formula $\log K = A + BX$, where K is the conductivity, X the potential gradient, and A and B are constants. Upon plotting X as abscissa and $\log K$ as ordinate, it is seen that for mica and glass at least the curves are nearly straight lines, though in the case of mica in the higher ranges of potential a curvature shows itself. Mica, when tested at different temperatures, presents curves which are parallel to one another, while for glass the slope of the curves changes in a systematic way with temperature. The influence of temperature on the resistivity of mica is enormous. For a certain specimen the resistivity for low potential gradients is 312 times as great at 13° C. as at 128°. Furthermore one specimen of mica shows a resistivity about 100 times that of another specimen, even when conditions are almost alike.

G. F. S.

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